Self-Reinforcing Market Dominance∗

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Abstract

Are initial competitive advantages self-reinforcing, so that markets exhibit an endogenous tendency to be dominated by only a few firms? Although this question is of great economic importance, no systematic empirical study has yet addressed it. Therefore, we examine experimentally whether firms with an initial cost advantage are more likely to invest in marginal cost reductions than firms with higher initial costs. We find that the initial competitive advantages are indeed self-reinforcing, but subjects in the role of firms overinvest relative to the Nash equilibrium. However, the pattern of overinvestment even strengthens the tendency towards self-reinforcing cost advantages relative to the theoretical prediction. Further, as predicted by the Nash equilibrium, mean-preserving spreads of the initial cost distribution have no effects on aggregate investments. Finally, investment spillovers reduce investment, and investment is higher than the joint-profit maximizing benchmark for the case without spillovers and lower for the case with spillovers.

Keywords: Cost-reducing Investment, Asymmetric Oligopoly, Increasing Dominance, Experimental Study

JEL Classification: C90, D43, L13, O31

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1 Introduction

In many markets, there are extended periods of time over which individual firms expand their lead and increasingly dominate the market. In some cases, market dominance results from exogenous sources such as state intervention, technology or demand shocks. Quite often, however, the dominance of a small number of firms or even a single market leader appears to be the endogenous outcome of market interaction. For example, Wal-Mart’s aggressive expansion resulted in market leadership within the U.S. discount retailing industry (Jia, forthcoming). Academic publishing has seen increasing concentration worldwide, with Elsevier leading the pack (Edlin and Rubinfeld, 2005). The ‘new economy’ also provides well-known examples of endogenous market dominance. For instance, Microsoft has acquired the lead in the markets for operating systems and office software, whereas Google dominates the market for search engines (Ferguson, 2005).

Obviously, an explanation of why market dominance came about in each of these examples requires detailed consideration of the particular case, each of which is characterized by idiosyncratic elements. Nevertheless, the examples lead to a common question: Are there any “natural” forces that explain why firms can so often maintain or even expand an initial lead, that is, why initial advantages of firms might be self-reinforcing, thereby leading to an extension of the initial lead?

To identify such a force in the simplest possible way, it is useful to consider a setting with a fixed number of firms that have different efficiency levels (different levels of marginal costs). Now suppose that the firms can invest into marginal cost reductions which increases both the output and the mark-up that they can command in product-market equilibrium. Typically, these two beneficial effects of lower marginal costs are mutually reinforcing: The higher mark-up is worth more when output is high, and conversely, the higher output is worth more when the mark-up is high. As a result of these demand-markup complementarities, firms that already have a high market share benefit more from an increase in their efficiency level than firms with a relatively low share, thus giving them a greater incentive to invest than their lagging competitors.

This kind of mechanism lies at the heart of most explanations of self-reinforcing dominance. In spite of the fact that convincing explanations for self-reinforcing dominance can be given, it is by no means true that there is a universal ten-

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1 For instance, in the Wal-Mart case, incremental investments and acquisitions of smaller firms played and important role. In academic publishing, there are mergers between big players. Microsoft benefited from network externalities, and Google introduced several successful product innovations.

2 Aydemir and Schmutzler (2008) identify similar forces in a setting where acquisitions and entry are allowed.

3 Similar arguments can be made for product quality improvements.

4 Atkey and Schmutzler (2001) make the point most explicitly, but the models of Flaherty (1980) and Budd et al. (1993) rely on similar forces. Similar effects are also present when a higher output involves lower costs because of learning-by-doing (Cabral and Riordan, 1994) or when it enhances demand because of network effects.
dency for markets to move in this direction. Commercial jet aircraft production, for instance, has seen several changes in market leadership since World War II (Sutton, 1998). In the PC market, IBM lost its initial dominance in the nineteen eighties (Stavins, 1995). These counterexamples are not necessarily evidence for a contradiction of theories of self-reinforcing dominance. Indeed, authors such as Athey and Schmutzler (2001) have pointed out that countervailing forces may well limit the power of increasing dominance. For instance, when imitation is cheap, catching up with a leader may be much less costly than expanding a lead. Then, even when demand-markup complementarities make cost reductions more attractive for leaders, the fact that any given cost reduction is easier to achieve for laggards may well mean that increasing dominance does not arise.

Unfortunately, in a given market environment, it is hard to be sure on purely theoretical grounds about whether economic fundamentals are indeed such that self-reinforcing dominance should emerge or whether countereffects should dominate. Moreover, markets are typically subject to many exogenous influences (that may favor or hinder increasing dominance), which makes it difficult to attribute the development of dominance to an endogenous self-reinforcing process.\(^5\)

To see whether economic agents respond to the incentives leading to self-reinforcing dominance, it is therefore important to control for potential exogenous shocks and, at the same time, to guarantee that the setting is such that the forces in favor of increasing dominance dominate over potential countervailing forces. Finding such a clean setting in real-world markets is difficult. Therefore, it is unsurprising that the empirical analysis of self-reinforcing dominance essentially reduces to anecdotal evidence. In laboratory experiments, however, it is possible to control for the above-mentioned confounding factors. In the following, we therefore present an experiment that tests whether demand-markup complementarities indeed lead to increasing dominance. In doing so, we provide, to our knowledge, the first experimental analysis of increasing dominance.\(^6\)

We study a simple version of a two-stage model of R&D competition that has received considerable attention in the literature. In this model, oligopolistic firms that potentially differ in their initial marginal costs first carry out cost-reducing investments which may or may not have positive spillover effects for the competitors. Then they engage in Cournot competition.\(^7\)

To address the issue of self-reinforcing dominance, we clearly require asymmetric treatments where subjects have different initial efficiency levels. In these asymmetric treatments, we assume that there are three types of firms, namely leaders, followers and laggards (in decreasing order of marginal costs). We compare the

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\(^5\)These exogenous influences may well be responsible for the fact that, in many industries, leadership changes emerge in the very long run.

\(^6\)Even beyond the issue of increasing dominance, the experimental analysis of R&D investment games is rare. Isaac and Reynolds (1998) and Suetens (2005) deal with issues of appropriability, Silipo (2005) and Zizzo (2002) investigate patent races.

\(^7\)Similar two-stage Cournot models have, for instance, been used by Brander and Spencer (1983), d’Aspremont and Jacquemin (1988), Suzumura (1992), and Leahy and Neary (1997).
investment levels of the three types of players in the asymmetric treatments. As the theoretical model underlying our experiment displays demand-markup complementarities, it predicts *increasing dominance*, the property whereby the more efficient firm tends to increase its lead by investing more into cost-reduction than the competitors.\(^8\) Technically, this prediction is an immediate implication of the structural properties of the game: (i) Actions (investments) are strategic substitutes, and (ii) a higher own state (efficiency level) and lower states of the competitors increase the marginal effect of the action on profits. In any game with these properties high-state firms choose higher actions, which implies increasing dominance in the specific context of our investment game (see Theorem 1 in Athey and Schmutzler, 2001). The evidence provides overwhelming support for the conclusion of this general result, and thus, in our specific context, for increasing dominance. Both in the spillover and in the no-spillover treatments, leaders invest more than followers and followers invest more than laggards. This result suggests that, in a dynamic context, market dominance would emerge endogenously, as small initial asymmetries would tend to reinforce each other. Importantly, increasing dominance comes out even more strongly in the lab than theory would suggest. Subjects have a tendency to overinvest relative to the Nash equilibrium, and this tendency is more pronounced the more efficient subjects initially are, that is, the lower their marginal costs. Thus the difference between the investments of more efficient and less efficient firms in the laboratory is greater than in the Nash equilibrium.

We vary treatments in three dimensions. In a first variation, we compare our asymmetric treatments with symmetric treatments where firms are initially identical. This allows us to address a fundamental issue in the analysis of R&D decisions, namely the relation between the “technological gap” (Aghion et al., 2001) and aggregate investment activity. Suppose one group of firms has lower marginal costs whereas another group has higher marginal costs, but the average efficiency is the same as in a reference situation with symmetric firms. Should aggregate investment in the latter “neck-to-neck” case be higher than in the former leader-laggard case?\(^9\) Theoretically, this is not obvious. On the one hand, with demand-markup complementarities, the leaders have higher investment incentives; on the other hand, the laggards have lower incentives, so that the aggregate effect is unclear. In the Cournot model underlying our analysis, the two effects exactly cancel out, so that investments are independent of the technological gap. Our experiments confirm the prediction, no matter whether we allow for spillovers or not.

A second variation concerns the appropriability of investments. We compare

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\(^8\)In the terminology of Athey and Schmutzler (2001), this would be *weak* increasing dominance. Note that we assume away potential countervailing forces by making the marginal costs of investment independent of previous investments.

\(^9\)The influence of the technological gap on investment incentives plays a central role in Aghion et al.’s (2001) analysis of the relation between competition and innovation. However, they consider a setting with differentiated price competition.
treatments where cost reductions have no spillovers on competitors with those where they do. This allows us to ask whether imperfect appropriability of investments indeed reduces subjects’ inclination to invest, as standard theory would predict (Spence, 1984). This is another theoretical result which has proved hard to confirm with field data: For instance, the disincentive effect of spillovers is not discernible in the data set employed by Levin (1988). He provides possible explanations why, contrary to the prediction of Spence’s model, investment is not discouraged by the high levels of spillover in electronics-based industries. Our experiments clearly show that lower appropriability reduces investments, which supports theory.\textsuperscript{10} The comparison of the spillover and the no-spillover treatment leads to another interesting observation. In both treatments, subjects overinvest relative to the Nash prediction. As investments have negative externalities in the no-spillover case and positive externalities in the spillover case, behavior is thus more cooperative than the Nash prediction suggests in the no-spillover case and less cooperative in the spillover case.\textsuperscript{11} In the paper, we suggest an explanation of this phenomenon that relies on social preferences.

Our third and final treatment variation is exclusively motivated by robustness considerations. We consider both low-efficiency and high-efficiency treatments, which differ in the initial average level of marginal cost. Our results are robust with respect to this treatment variation.

A striking feature of our analysis is that the deviations from the Nash equilibrium are, albeit significant, fairly small. In fact, given the complex nature of the experiment, it is surprising how close the outcome is to the Nash equilibrium, no matter whether we are considering treatments with or without spillovers, and symmetric or asymmetric cases.

The remainder of the paper is organized as follows. Section 2 introduces the analytical framework. Section 3 formulates the testable hypotheses. Section 4 describes the experimental design. Section 5 contains the results. Section 6 concludes.

2 Analytical Framework

The analytical framework that we require for the experimental analysis combines features of the two-stage model of d’Aspremont and Jacquemin (1988) with the dynamic analysis of Athey and Schmutzler (2001): The ex-ante heterogeneity between firms that is central to the latter paper is introduced into the static framework of d’Aspremont and Jacquemin.

Our model is deliberately designed to capture the essence of the strategic interaction in investment models. Like d’Aspremont and Jacquemin, we there-\textsuperscript{10} Suetens (2005) also shows that lack of appropriability has negative effects on investment, but she only considers a symmetric Cournot duopoly setting. Isaac and Reynolds (1988) have a similar result in a stochastic invention model.

\textsuperscript{11} Andreoni (1995) comes to similar conclusions for privately provided public goods.
fore consider only one period of investment. Even though this might seem to be at odds with our objective of understanding important aspects of dynamic investment behavior, there are several reasons why we proceeded in this fashion. First, most importantly, the basic forces towards high investments of relatively efficient firms that show up in a fully dynamic model are already present in a one-period version of the model, so that the subgame perfect equilibrium of the dynamic game also satisfies increasing dominance with respect to initial efficiency levels. Intuitively, in the static version leaders invest more because they benefit more from demand/markup-increases; in the dynamic version they also take the effects of their investments in future rounds of the investment game into account. However, these long-term considerations reinforce the short-term considerations, because any given improvement of the initial position in a future investment game is more valuable for a firm that starts out ahead of the others. Second, while a dynamic version of the game is implementable in principle, the strategic complexity of the situation is likely to lead to informational overload in an experimental context. Third, our approach of considering only one period per game allows us to obtain more observations.

2.1 Setup

We consider an oligopolistic industry with a finite number of $I \geq 2$ firms producing a homogeneous product. Let $p = a - Q$ be the inverse demand function, where $p$ and $Q$ denote, respectively, the price and the aggregate output of this product. Firms engage in two-stage competition. In the first stage, each firm $i$ chooses an investment in marginal cost reduction. In the second stage, firms compete à la Cournot in the product market.

We assume that firm $i$ initially has marginal cost $c - Y^i_0$ for some exogenous reference level $c$ of marginal costs in the industry, so that $Y^i_0$ is interpreted as the initial (exogenous) efficiency level of firm $i$. In the first stage, given $Y_0 \equiv (Y^1_0, ..., Y^I_0)$, each firm $i$ takes an investment decision, $y^i$, and we let $y \equiv (y^1, ..., y^I)$. In the second stage, firm $i$ has marginal costs

$$c^i = c - Y^i_1,$$  \hspace{1cm} (1)

where $Y^i_1$ is the efficiency level at the beginning of this stage.

Firm $i$’s efficiency level at the beginning of the product market stage depends on its initial efficiency level, on own investment, and possibly also on each competitor’s investment in marginal cost reduction. More specifically, firm $i$’s efficiency level is

$$Y^i_1 = Y^i_0 + y^i + \lambda \sum_{j \neq i} y^j,$$  \hspace{1cm} with  \hspace{1cm} $\lambda \in [0, 1].$  \hspace{1cm} (2)

Here, the parameter $\lambda$ captures spillovers at the industry level, that is, it provides an inverse measure of the overall level of appropriability. If $\lambda = 0$, there are
no spillovers, whereas if \( \lambda = 1 \), each firm’s investments are shared completely. Obviously, for \( 0 < \lambda < 1 \), the spillovers are imperfect.\(^{12}\)

The investment cost function for a direct reduction in marginal costs is given by

\[
k(y^i) = \kappa (y^i)^2, \quad \text{with} \quad \kappa > 0.
\]

Thus, the function displays increasing marginal costs.\(^{13}\)

When firms choose their investments \( y \) in the first stage, they can either do so non-cooperatively or cooperatively. In both cases, we solve the game using backward induction.

### 2.2 The Second Stage

At the beginning of the second-stage game, \( Y_1 \equiv (Y^1_1, ..., Y^I_1) \) summarizes the firms’ efficiency levels, which correspond to marginal costs \( c \equiv (c^1, ..., c^I) \). It is well known that equilibrium outputs in the linear Cournot model with heterogeneous firms are given by

\[
q^i(c) = a - Ic^i + \frac{\sum_{j \neq i} c^j}{I + 1}.
\]

Substituting \( c^i = c - Y^i_1 \) from (1) and letting \( \alpha \equiv a - c \) denote the net-demand parameter, equilibrium output levels as a function of efficiency levels can be expressed as

\[
q^i(Y_1) = \frac{\alpha + IY^i_1 - \sum_{j \neq i} Y^j_1}{I + 1}.
\]

An immediate implication is that equilibrium product market profits are given by

\[
\pi^i(Y_1) = \left( \frac{\alpha + IY^i_1 - \sum_{j \neq i} Y^j_1}{I + 1} \right)^2. \tag{3}
\]

### 2.3 The First Stage

Equation (3) gives the equilibrium product market profits in the second-stage game as a function of the first-stage outcome, summarized by \( Y_1 \). To obtain an expression for firm \( i \)'s net profit in terms of cost reductions and the parameters, we substitute the efficiency levels by the corresponding expression given in (2)

\(^{12}\)In the duopoly case, Eq. (2) includes the model of d’Aspremont and Jacquemin (1988) if the firms’ initial efficiency levels are zero. In the more general framework of Athey and Schmutzler (2001), Eq. (2) provides a simple explicit specification of firm \( i \)'s state dynamics.

\(^{13}\)Note that this cost function depends only on investments and not on initial efficiency levels.
and subtract the costs of investing. After rearranging, firm $i$’s net profit reads

$$\Pi^i(y; Y_0, \alpha, \lambda, \kappa) = \left( \frac{\alpha + IY^i_0 - \sum_{j \neq i} Y^j_0 + (I + \lambda(1 - I)) y^i + (2\lambda - 1) \sum_{j \neq i} y^j}{I + 1} \right)^2 - \kappa (y^i)^2. \quad (4)$$

Assuming positive outputs, differentiating firm $i$’s net profit with respect to $y^j$ implies

$$\text{sign} \left( \frac{\partial \Pi^i}{\partial y^j} \right) = \text{sign} (2\lambda - 1),$$

which gives rise to the following observation:

**Observation 1.** The game is characterized by negative (positive) externalities if the spillover parameter $\lambda$ is smaller (larger) than 0.5, as a marginal increase of a rival’s investment reduces (increases) firm $i$’s net profit.

To understand this observation, note that an increase in the investment of a competitor affects a firm through two channels. First, there is a negative effect of facing a more efficient competitor. Second, there is a positive effect of becoming more efficient by obtaining spillovers. For $\lambda < 0.5$, the negative effect dominates; for values of $\lambda > 0.5$, the positive effect does.

We now proceed to determine the subgame-perfect equilibrium investments.

### 2.4 The Subgame-Perfect Equilibrium

Assuming that the firms choose their investments non-cooperatively, firm $i$’s optimal investment decision, taking the decisions of the other firms $y^{-i}$ as given, solves

$$\max_{y^i \geq 0} \Pi^i(y^i, y^{-i}; Y_0, \alpha, \lambda, \kappa).$$

In the subsequent analysis, we proceed under the assumption that second-order and stability conditions hold.\textsuperscript{14} Reflecting the quadratic objective function, firm $i$’s best-response function is linear and shown in the Appendix to be of the form

$$R^i(y^{-i}) = \phi^i - \frac{\Pi_{ij}^i}{\Pi_{ii}^i} \sum_{j \neq i} y^j, \quad \text{with} \quad \phi^i > 0,$$

where subscripts denote partial derivatives. Note that firm $i$’s output depends only on the sum of the opponents’ outputs.\textsuperscript{15} This property of the Cournot game allows us to present the game to the subjects in matrix form.

\textsuperscript{14}A formal statement of these conditions is provided in the Appendix.

\textsuperscript{15}Moreover, using the second order condition,

$$\text{sign} \left( \frac{\partial R^i(y^{-i})}{\partial y^j} \right) = \text{sign} (\Pi_{ij}^i) = \text{sign} (2\lambda - 1),$$
In the Appendix, we provide a closed-form solution for the equilibrium investment levels. Substituting these quantities in the corresponding expression above produces firm $i$’s equilibrium output level, the product market profit, and the net profit attained in equilibrium.

2.5 Implications

In this section, we shall derive three testable implications of the theory. The first result addresses increasing dominance and shows that more efficient firms invest more in equilibrium than their competitors. The proposition thus provides the core of an argument for self-reinforcing concentration.\(^{16}\)

**Proposition 1** (Dominance). For all $i \neq j$, $Y^i_0 > Y^j_0$ implies $y^i_0 > y^j_0$.

*Proof.* See the Appendix. □

To understand the intuition, it is important to note two key properties of firm $i$’s net profit function given in (4), namely

$$\frac{\partial^2 \Pi^i(\cdot)}{\partial Y^i_0 \partial y^i} > 0 \quad \text{and} \quad \frac{\partial^2 \Pi^i(\cdot)}{\partial Y^j_0 \partial y^i} < 0.$$ (5)

The first property is very intuitive once one takes the underlying oligopoly model into account: Other things equal, firms with high initial efficiency level $Y^i_0$ have high demand (mark-up). The profit gain from increasing mark-up (demand) by investing into marginal cost reduction is therefore higher. This property suggests that, *leaving strategic effects aside*, firms with high initial efficiency levels should invest more than firms with low initial efficiency levels. Similar reasoning can be used to explain the second property intuitively: This property implies that firms invest more when competitors have low initial efficiency levels. Together, both properties identify the source of increasing dominance: The high demand of a leader coming from its high initial efficiency level and the competitor’s low initial efficiency level both increase the marginal incentive to invest.\(^{17}\)

The next result is an immediate consequence of the quadratic net profit function.

**Proposition 2** (Technological Gap). (i) Given $Y_0$, aggregate investment $y^* = \sum_i y^*_i$ is determined by the sum of the initial efficiency levels, $\sum_i Y^i_0$, independently of their distribution. (ii) For an asymmetric initial efficiency profile $Y_0$, so that reaction curves are downward sloping if $\lambda < 0.5$ (i.e., investments are strategic substitutes) and upward sloping if $\lambda > 0.5$ (i.e., investments are strategic complements). If $\lambda = 0.5$, the firms’ investment choices are independent of rivals’ actions. The proof of the last equality is provided in the Appendix.

\(^{16}\)Increasing dominance can also be derived from more general results in Athey and Schmutzler (2001); the remaining results we provide are novel for the asymmetric case.

\(^{17}\)Propositions 1 and 2 in Athey and Schmutzler (2001) make this intuition more precise by showing that the properties given in (5) imply increasing dominance both in the case of strategic substitutes and in the case of strategic complements.
and a symmetric profile \( Y_0^S \) with the same sum of initial efficiency levels, the most efficient type in \( Y_0 \) invests more than each firm in the symmetric profile and the least efficient type invests less.

Proof. See the Appendix. \( \square \)

The result implies that in the specific setting of the linear Cournot model with quadratic investment costs, increasing asymmetry of firms has no effects on their aggregate investments. Higher incentives to invest for more efficient firms are exactly offset by lower incentives for less efficient firms. In other words, neck-to-neck competition and leader-laggard structures lead to the same aggregate investment.

The third result shows that decreasing appropriability reduces investments.

**Proposition 3 ( Appropriability).** Suppose that the following condition holds:

\[
\frac{\partial^2 \Pi_i(\cdot)}{\partial \lambda \partial y_i} < 0. \quad (6)
\]

Then, for any pair \( \lambda', \lambda'' \) such that \( \lambda' < 0.5 < \lambda'' \) and every \( i \), \( y^*_{\lambda'}(\lambda') > y^*_{\lambda''}(\lambda'') \).

Proof. See the Appendix. \( \square \)

Intuitively, an increase in \( \lambda \) has two countervailing effects on marginal investment incentives. First, higher spillovers mean that investments have a stronger positive effect on the competitor’s efficiency, which makes investment less attractive. Second, however, for given cost reductions of the competitors, larger values of the spillover parameter reduce firm \( i \)’s marginal cost and thus increase its efficiency level. The resulting increase in demand (mark-up) then leads to a higher investment of firm \( i \). Condition (6) ensures that the first of the two effects dominates, so that a higher value of the spillover parameter reduces firm \( i \)’s marginal incentive to invest.\(^{18}\)

### 2.6 The Cooperative Benchmark

As a benchmark for the non-cooperative game, we now consider the model where firms choose outputs non-cooperatively, but choose investments so as to maximize their joint profit.\(^{19}\)

\(^{18}\)Straightforward calculations show that condition (6) is automatically satisfied in symmetric games and met in asymmetric games if and only if firms are not “too asymmetric”. In the experimental specification, the parameters are chosen to meet the requirement.

\(^{19}\)For the symmetric case, the cooperative benchmark is often regarded as an appropriate description of R&D-cartels, where firms are allowed to cooperate in R&D, but must compete on the product market (see, e.g., d’Aspremont and Jacquemin, 1988).
Assuming that the firms pick their investments cooperatively, the problem is to

\[
\max_{y \in \mathbb{R}^I} \Pi(y; Y_0, \alpha, \lambda, \kappa) = \sum_{i=1}^{I} \Pi^i(y; Y_0, \alpha, \lambda, \kappa)
\]

s.t. \( y^i \geq 0, \quad i = 1, \ldots, I. \)

Assuming that the Hessian of the joint-profit function is negative definite, a unique solution exists. We refrain from characterizing the solution analytically and evaluate it numerically for our experimental study.\(^{20}\)

The final result allows us to compare firm \( i \)'s non-cooperative investment decision to firm \( i \)'s optimal investment choice under a cooperative agreement.

**Proposition 4** (Deviation from JPM). Let \( y^* \) and \( y^{**} \) denote firm \( i \)'s equilibrium investment levels under non-cooperation and cooperation, respectively, and suppose that \( \Pi(\cdot) \) is concave. For \( \lambda < 0.5 \), we have \( y^*(\lambda) > y^{**}(\lambda) \); for \( \lambda > 0.5 \), we have \( y^*(\lambda) < y^{**}(\lambda) \).

**Proof.** See the Appendix. \( \square \)

The intuition reflects the nature of the externality from investment. For illustration, consider the case where \( \lambda < 0.5 \), where investing exerts a negative externality on rival firms. Therefore, players invest more than socially optimal for the group of players and the equilibrium investment lies above the joint-profit maximizing level.

## 3 Hypotheses

We now summarize the testable hypotheses that the theory provides. The following Hypotheses 1 through 4 are implications of Nash behavior. If they are confirmed in the laboratory setting, findings are consistent with the view that the rational choice model of Section 2 captures important aspects of subjects’ behavior.

The first hypothesis corresponds to Proposition 1.

**Hypothesis 1** (Dominance). In asymmetric games, firms with a higher initial efficiency level invest more than firms with a lower initial efficiency level.

Next, we turn to the two comparative-statics predictions. Proposition 2 yields the following hypothesis.

**Hypothesis 2** (Technological Gap). Changes in the distribution of initial efficiency levels have no impact on aggregate investments, but compared to the symmetric case, a mean-preserving spread of the initial efficiency levels leads to higher investments of the leader and lower investments of the laggard.

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\(^{20}\)Given a set of parameters, it is easy to check whether the Hessian is negative definite.
Thus, in the specific setting discussed here, it does not matter for the aggregate investment whether competition is neck-to-neck or if firms with high marginal costs face competitors with low marginal costs.

Proposition 3, which reflects the notion that decreasing appropriability reduces investments, leads to the following hypothesis.

**Hypothesis 3** (Appropriability). *As spillovers increase, players invest less.*

Our final derived hypothesis concerns the players’ deviation from the joint-profit maximization benchmark.

**Hypothesis 4** (Deviation from JPM). *Relative to joint-profit maximization, players overinvest (underinvest) in the presence of negative (positive) externalities.*

Hence, we expect players to deviate in precisely the way that one would expect from rational players in settings with positive and negative externalities, respectively.

### 4 Experimental Design

#### 4.1 Overview

The experiment was designed to investigate Hypotheses 1 through 4. To test Hypothesis 1, we require asymmetric treatments, where firms differ in their initial efficiency levels. To test Hypothesis 2, we compare such asymmetric treatments (ASYM) with symmetric treatments (SYM) where all firms have identical initial efficiency levels, but the average efficiency is the same. To test Hypotheses 3 and 4, we compare no-spillover treatments (NS) with spillover treatments (S).

Finally, we introduce a third dimension of treatment variation for robustness considerations: We compare a setting where firms have high initial efficiency levels (HE) with one where they have low initial efficiency levels (LE).

Table 1 summarizes our treatments, highlighting the three dimensions of treatment variation. As we shall detail in Section 4.2, we varied the spillover dimension across subjects and the two other dimensions within subjects. More specifically, we chose the parameter values as follows. In all treatments, groups of six players were formed, possible investment choices were restricted to the interval \([0, 12]\), and the net-demand parameter \((\alpha = 120)\) and the cost parameter \((\kappa = 3)\) were unaltered. We chose \(\lambda = 0\) in the no-spillover treatments and \(\lambda = 0.6\) in the spillover treatments.\(^{21}\) These choices guarantee that we have negative externalities (and strategic substitutes) in the no-spillover case, and positive externalities (and strategic complements) in the spillover case. Finally, we chose the initial efficiency levels as shown in Table 2. The levels are the same.

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\(^{21}\)The parameter value \(\lambda = 0.6\) was chosen to make the calculations for the subjects relatively simple.
Table 1: Summary of treatments.

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<td>ASYM-HE</td>
<td>ASYM-HE</td>
</tr>
</tbody>
</table>

Notes: SYM and ASYM refer to symmetric and asymmetric treatments, respectively. LE and HE indicate settings with low and high efficiency levels.

in the spillover and no-spillover case. For the symmetric treatments, we chose $Y_0 = (5.5, ..., 5.5)$ and $Y_0 = (11, ..., 11)$. In the corresponding asymmetric treatments, $Y_0 = (9, 9, 6.5, 6.5, 1, 1)$ and $Y_0 = (18, 18, 13, 13, 2, 2)$, respectively. In particular, there are three types of players, “leaders” (highest efficiency), “followers” (medium efficiency) and “laggards” (lowest efficiency).

4.2 Details

To implement our treatment variations, we chose to vary the spillover dimension across subjects, whereas the other two dimensions where varied within subjects only. We confronted subjects with eight different roles, with each role repeated twice. Subjects played the symmetric high-efficiency and low-efficiency treatments, and they took the role of the leaders, laggards and followers in the asymmetric high-efficiency and low-efficiency treatments, respectively. Thus, four out of the sixteen investment decisions relate to symmetric treatments, whereas the remaining ones reflect outcomes in asymmetric treatments.²²

In the two-stage model of Section 2 firms first choose their investment levels and then they compete in the product market. In order to isolate the impact of the incentives arising from the investment stage in the cleanest possible way we confronted subjects at the investment stage with product market profit tables that were based on the Cournot equilibrium that results from every efficiency

²²Observe that we could have carried out the asymmetric treatments with only two types of players. However, our approach allows us to check whether the behavior of leaders relative to followers differs from the behavior of followers relative to laggards.

²³Specifically, in both the spillover and the no-spillover treatment, two sessions with five six-player groups were run.
Table 2: Distributions of initial efficiency levels.

<table>
<thead>
<tr>
<th>Within Subjects Treatments</th>
<th>LE</th>
<th>HE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYM</td>
<td>$Y_0 = (5.5, ..., 5.5)$</td>
<td>$Y_0 = (11, ..., 11)$</td>
</tr>
<tr>
<td>ASYM</td>
<td>$Y_0 = (9, 9, 6.5, 6.5, 1, 1)$</td>
<td>$Y_0 = (18, 18, 13, 13, 2, 2)$</td>
</tr>
</tbody>
</table>

Notes: SYM and ASYM refer to symmetric and asymmetric treatments, respectively. LE and HE indicate settings with low and high efficiency levels.

combination at the end of the investment stage. Thus, subjects did not choose the output quantities at the second stage of the game; they merely determined their investment levels at the first stage, knowing the consequences of each combination of their own efficiency level and the efficiency level of the other members of the group.

This simplified approach plays a central role in identifying the sources of deviations from the Nash equilibrium in the investment stage. To see this, consider the alternative setting where players actually choose outputs after they have observed investments. The evidence from Cournot experiments suggests that average output choices in the product-market stage would be consistently above the Nash equilibrium.\(^{24}\) Now suppose, in the investment stage, subjects behave according to this prediction. Then clearly they would have an incentive to overinvest relative to the Nash equilibrium that comes exclusively from their anticipation of the second-stage outcome. So if we observed excessive investment levels, they might therefore result from correctly anticipated second-period deviations or from alternative motives to choose other investments. Focusing on the investment stage alone allows us to exclude the first possibility.

Irrespective of the treatment, each replication of the static game is described as follows. At the beginning of each replication, the experimenter informs subjects about the initial efficiency level of the firm they are representing, and the initial efficiency levels of the other firms in their group. Then, they can choose investment levels which improve their initial efficiency level. Each subject knows that its product market profit depends both on the own efficiency level and the

\(^{24}\)In treatments with symmetric firms, the experiments of Huck et al. (2004) as well as their survey suggest that total average output often exceeds the Nash prediction in markets with three or more firms. For firms with asymmetric costs, which is more relevant in our case, several papers report similar results even in duopolies (Mason et al., 1992; Mason and Phillips, 1997; Rassenti et al., 2000).
Table 3: Part of the product market profit table.

<table>
<thead>
<tr>
<th>$Y_i^1$ →</th>
<th>5.5</th>
<th>6.0</th>
<th>...</th>
<th>9.5</th>
<th>10.0</th>
<th>10.5</th>
<th>11.0</th>
<th>11.5</th>
<th>12.0</th>
<th>12.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{Y}_{i-1}$ ↓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>321</td>
<td>337</td>
<td>...</td>
<td>456</td>
<td>475</td>
<td>493</td>
<td>513</td>
<td>532</td>
<td>552</td>
<td>573</td>
</tr>
<tr>
<td>6.0</td>
<td>309</td>
<td>324</td>
<td>...</td>
<td>441</td>
<td>459</td>
<td>478</td>
<td>497</td>
<td>516</td>
<td>536</td>
<td>556</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10.0</td>
<td>217</td>
<td>229</td>
<td>...</td>
<td>329</td>
<td>345</td>
<td>361</td>
<td>377</td>
<td>394</td>
<td>412</td>
<td>429</td>
</tr>
<tr>
<td>10.5</td>
<td>206</td>
<td>219</td>
<td>...</td>
<td>316</td>
<td>332</td>
<td>348</td>
<td>364</td>
<td>380</td>
<td>397</td>
<td>414</td>
</tr>
<tr>
<td>11.0</td>
<td>196</td>
<td>208</td>
<td>...</td>
<td>304</td>
<td>319</td>
<td>334</td>
<td>350</td>
<td>366</td>
<td>383</td>
<td>400</td>
</tr>
<tr>
<td>11.5</td>
<td>186</td>
<td>198</td>
<td>...</td>
<td>291</td>
<td>306</td>
<td>321</td>
<td>337</td>
<td>353</td>
<td>369</td>
<td>386</td>
</tr>
<tr>
<td>12.0</td>
<td>177</td>
<td>188</td>
<td>...</td>
<td>279</td>
<td>294</td>
<td>309</td>
<td>324</td>
<td>340</td>
<td>356</td>
<td>372</td>
</tr>
<tr>
<td>12.5</td>
<td>167</td>
<td>178</td>
<td>...</td>
<td>268</td>
<td>282</td>
<td>296</td>
<td>311</td>
<td>327</td>
<td>342</td>
<td>358</td>
</tr>
</tbody>
</table>

Notes: $Y_i^1$ and $\bar{Y}_{i-1}$ denote, respectively, the own efficiency level and the competitors’ average efficiency level. The best replies (in italics) and the equilibrium product market profit (in bold face) are highlighted for illustrative purposes.

average efficiency level of the other players in the group. Since the subjects choose their investment simultaneously, they do not yet know the average ex-post efficiency level of the other members in their group when they make their choices; thus, they have to form expectations about their competitors’ average efficiency level. To calculate the payoffs corresponding to these expectations and on their own investment decisions, subjects can use product market profit and cost tables, as well as a calculator.\textsuperscript{25}

To illustrate the product market profit table, we now consider the treatment \textit{NS-LE-ASYM}. Table 3 shows a part of the table that the subjects used.\textsuperscript{26} The first row gives the efficiency level of the subject’s firm, whereas the first column gives the average efficiency level of the other firms. For example, the subjects’ product market profit is 573 points for $Y_i^1 = 12.5$ and $\bar{Y}_{i-1} = 5.5$. Thus, subject $i$’s product market profit is 573 points after investing 7 units (as the initial efficiency level is 5.5 units), under the assumption that the other firms do not invest. To obtain their net profit, subjects used the cost table, which is presented in part in Table 4. Using the table, subjects could find out the relevant cost of an investment equal to 7 units (which are $3 \times 7^2 = 147$ points). In this fashion, subjects could, in principle, compute the best reply for a given expectation of average investments of the other firms.

\textsuperscript{25}We chose to present two separate tables to subjects to highlight the nature of the game as involving costly investments that influence product market profits.

\textsuperscript{26}By construction, the product market profit table conditions on the firms’ efficiency levels at the beginning of the product market stage. Thus, the same table can be used in both treatments.
<table>
<thead>
<tr>
<th>Investment</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>1.0</td>
<td>3.00</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>11.5</td>
<td>396.75</td>
</tr>
<tr>
<td>12.0</td>
<td>432.00</td>
</tr>
</tbody>
</table>

Notes: Investment costs result from the cost function $k(y^i) = 3 (y^i)^2$, where $y^i$ denotes firm $i$’s investment level.

After 90 seconds, the subjects must take a definite decision. Finally, at the end of each replication of the static game, subjects are informed about actual investments of each group member and their own net profit.

In all $S$-treatments, the subjects’ tasks are exactly the same. The only difference to the $NS$-treatments arises due to the presence of spillovers: The subjects’ efficiency levels are not determined by the sum of their own initial efficiency level and their own investment only—they also depend on the spillovers from group members.\(^{27}\)

Subjects were recruited using ORSEE (Greiner, 2004), and were randomly allocated to groups of six subjects upon arrival at the laboratory (partners setting).\(^{28}\) Subjects were students from the University of Zurich and the Swiss Federal Institute of Technology in Zurich. A total of 120 subjects participated in the experiment, and none of them in more than one session. All experiments were computerized using the software “z-Tree” (Fischbacher, 2007) to run the experiment.

Before subjects played the experiment, they were given time to carefully read the instructions and to solve some simple examples to make sure that they understood the experiment correctly. There was no communication during the experiment.

An average session lasted 120 minutes. The net profits in points attained in the 16 replications of the games were converted to Swiss francs (1 point = CHF 0.80). On average, a subject earned CHF 46.55 (about $38) in the $NS$-treatment and CHF 59.85 (about $49) in the $S$-treatment, including a show-up fee of CHF 10.00 (about $8).

\(^{27}\)Specifically, subjects know that their own efficiency level is determined by the sum of their own initial efficiency level, own investment, and three times the average investment of the other group members. Formally, this can be seen by letting $\lambda = 0.6$ in Eq. (2).

\(^{28}\)To avoid the potential of reputation building, each group member’s group member’s position on its subject screen was changed in every period, and subjects were informed of this.
Table 5: Theoretical benchmarks and summary of investment decisions.

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Benchmarks</th>
<th>Experimental Investments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nash</td>
<td>JPM</td>
</tr>
<tr>
<td>No-Spillover</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SYM-LE</td>
<td>Type 5.5</td>
<td>5.34</td>
</tr>
<tr>
<td></td>
<td>Type 11</td>
<td>5.57</td>
</tr>
<tr>
<td>SYM-HE</td>
<td>Type 9</td>
<td>6.74</td>
</tr>
<tr>
<td></td>
<td>Type 6.5</td>
<td>5.74</td>
</tr>
<tr>
<td></td>
<td>Type 1</td>
<td>3.54</td>
</tr>
<tr>
<td>ASYM-LE</td>
<td>Type 18</td>
<td>8.37</td>
</tr>
<tr>
<td></td>
<td>Type 13</td>
<td>6.37</td>
</tr>
<tr>
<td></td>
<td>Type 2</td>
<td>1.97</td>
</tr>
<tr>
<td>ASYM-HE</td>
<td>Type 18</td>
<td>3.97</td>
</tr>
<tr>
<td></td>
<td>Type 13</td>
<td>3.21</td>
</tr>
<tr>
<td></td>
<td>Type 2</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Notes: Average and median experimental investments, standard deviations (S.D.), and average deviations from the Nash equilibrium (∆) based on 120 observations for each firm type.

5 Experimental Results

This section presents tests of Hypotheses 1 through 4, which are all implied by Nash behavior. In Section 5.1, we shall first compare the experimental investment decisions to the Nash benchmarks. It will turn out that there is significant overinvestment relative to the Nash equilibrium, so that we cannot take the hypotheses for granted. In the remaining Sections 5.2 through 5.5, we therefore test each hypothesis in turn.
Table 6: Overinvestment.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep. Var.: $\Delta_{i,t,k}$</td>
<td>Dep. Var.: $\Delta_{i,t,k}^{C}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>const</td>
<td>0.6372***</td>
<td>0.8083***</td>
<td>4.5394***</td>
</tr>
<tr>
<td></td>
<td>(0.0943)</td>
<td>(0.1197)</td>
<td>(0.1309)</td>
</tr>
<tr>
<td>late</td>
<td>$-0.3421^{* ***}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1035)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>spill</td>
<td></td>
<td>$-4.9720^{* ***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1830)</td>
<td></td>
</tr>
<tr>
<td>sym</td>
<td></td>
<td>0.4152***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1007)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Overinvestment (Models I and II, respectively), and the observed deviation from the JPM prediction (Model III) at the overall level. Dependent variable in Models I and II is subject $i$’s period $t$ overinvestment in group $k$; in Model III, dependent variable is subject $i$’s period $t$ deviation from the JPM prediction in group $k$. 360 observations in each treatment; *** = Significant at the 1% level. Robust standard errors adjusted for clustering on groups in parenthesis.

5.1 The Predictive Power of Nash Benchmarks

Table 5 presents some simple summary statistics of experimental investment decisions, along with the theoretical predictions. A comparison of average type-specific investments and Nash benchmarks suggests, except for type-1 firms, a tendency to overinvest relative to the Nash prediction (in Table 5, the corresponding deviations from the Nash equilibrium, $\Delta$, are positive).

To test whether the deviation from the Nash prediction is statistically significant, we first introduce some notation. We let $\hat{y}_{i,t,k}$ denote subject $i$’s period $t$ investment decision, where the subscript $k$ assigns the observation to the group, or industry, in which the subject operates. Similarly, we let $y_{i,t}^*$ denote the Nash prediction. The overinvestment relative to the Nash prediction, $\Delta_{i,t,k}$, can thus be expressed as

$$\Delta_{i,t,k} = \hat{y}_{i,t,k} - y_{i,t}^*.$$ 

Regressing overinvestment on a constant yields an estimate of 0.64 units with (robust) standard error 0.094 (see Table 6, Model I). Thus, there is a highly significant overall tendency to overinvest relative to the Nash benchmark. This result is also supported when we split the sample into early and late periods (see Table 6, Model II): Although overinvestment significantly decreases by 0.34

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29 Standard errors are clustered on groups as within-group observations may not be independent of each other (both in a given replication and over replications).
30 Throughout the paper, early and late periods refer to the first and second replication of
units from 0.81 to 0.47 units in late periods, it remains persistent at the 1% confidence level. Summing up, we have the following:

**Observation 2.** There is significant, albeit small, overinvestment relative to the Nash benchmark that varies across treatments.

It is important to note that this observation is essentially independent of the treatment and of the role (leader, follower, laggard) that an individual plays: In 14 of the 16 cases, the players’ investments are (slightly) above the Nash prediction. In particular, overinvestment occurs in both the no-spillover and the spillover treatments. This is striking, because overinvestment in a game without spillovers corresponds to a behavior that is less cooperative than in the Nash equilibrium, whereas in the game with spillovers the overinvestment corresponds to more cooperative behavior.

A possible explanation for this phenomenon relies on the fact that investments are strategic complements in the case with spillovers but substitutes in the case without spillovers. This difference in strategic incentives between spillover and no-spillover treatments could interact with the existence of reciprocal preferences such that overinvestment results in both cases.

Suppose, for example, that the population contains reciprocal and egoistic players and assume that the reciprocal players expect others to overinvest in the spillover treatment. Overinvestment in the spillover treatment means that the overinvesting players generate a benefit (positive externality) for the others, i.e., overinvesting is a kind behavior. Therefore, a reciprocal player will respond to this expectation with overinvestment. In addition, the selfish players will also overinvest because of strategic complementarity (i.e., they have pecuniary incentives to overinvest given that the reciprocal players overinvest relative to Nash). Thus, in the case of spillovers between the investing subjects, strategic complementarity and a positive fraction of reciprocal players may contribute to overinvestment.

1. Each HE and LE treatment, respectively, as subjects are twice in each role.
2. The result is corroborated at the treatment level when controlling for spillovers and symmetry.
3. The role of strategic complementarity and substitutability for aggregate deviations from rationality or Nash equilibrium play has been examined by Haltiwanger and Waldmann (1985, 1989), Fehr and Tyran (2001, 2005), and Potters and Suetens (2005).
4. Invoking the existence of players with social preferences seems justified because there is ample evidence that such preferences may play a role in strategic games in which players can affect each others payoffs (Fehr and Schmidt, 2006). Often social preferences take the form of preferences for reciprocity (Levine, 1998; Rabin, 1993; Duwenberg and Kirchsteiger 2004; Falk and Fischbacher, 2006; Cox et al., 2007; Cox et al., 2008). An individual with reciprocal preferences responds to (the expectation of) kind acts with kind behavior and to (the expectation of) hostile behavior with hostility.
5. There is considerable evidence from public good games (e.g., Fischbacher et al., 2001; Kocher et al., 2008) that a substantial share of the players are willing to contribute to the public good if they believe that the other players are also contributing.
In the no-spillover case the existence of reciprocal players may contribute to overinvestments relative to the Nash equilibrium because in these treatments investing imposes a negative externality on the other subjects. Therefore, investing according to or above the Nash equilibrium is likely to be viewed by the reciprocal subjects as unkind behavior that deserves retaliation whereas underinvestment relative to the Nash equilibrium is likely to be viewed as kind behavior because it reduces negative externalities. Note also, that due to strategic substitutability, egoistic players will never reciprocate kind acts of underinvestment; instead, they respond to underinvestment with overinvestments. Whether the selfish players play the Nash equilibrium or whether they even overinvest, reciprocal players are likely to interpret such behaviors as hostile and respond with retaliation, i.e., they will overinvest in order to punish the other investors.\textsuperscript{35} Thus, overinvestment in the no-spillover treatment could be the result of reciprocal players’ retaliatory behavior.\textsuperscript{36}

Because investments are close to the levels prescribed by the Nash hypothesis, it seems conceivable that Hypotheses 1 through 4 will be confirmed. Nevertheless, as observations and predictions differ, we cannot take this for granted.

5.2 Increasing Dominance

To investigate our main hypothesis of increasing dominance, we now compare the investment behavior in the asymmetric treatments. We have the following result:

**Result 1a (Dominance).** The higher a firm’s initial efficiency level the larger is the firm’s investment on average, that is, subjects’ behavior exhibits increasing dominance. This result holds in each of the asymmetric treatments.

Figures 1 and 2 provide a graphical representation of Result 1a (for treatments \textit{NS} and \textit{S}, respectively): On average, the more efficient firms invest more than less efficient firms.\textsuperscript{37} This notion can be confirmed in Figure 3, which goes beyond Figures 1 and 2 by plotting the cumulative distribution of the subjects’ investment choices instead of averages only. Inspection of the figure reveals that the cumulative distribution function of investments of leaders (followers) is below

\textsuperscript{35}Recall that in the no-spillover treatment investing implies imposing a negative externality on the other players. There is evidence that in games with negative externalities cooperation is considerably more difficult to sustain (Andreoni, 1995). This is consistent with the notion that in an environment with negative externalities mutual hostility is more likely to prevail. In the no-spillover treatment, mutual hostility implies overinvestment.

\textsuperscript{36}The tendency to overinvest is related to results in Huck et al. (2004). Although the authors analyze the Cournot game only, there are important structural similarities between this game and our stage-game in the no-spillover treatment: Both games involve negative externalities and strategic substitutes. It is therefore interesting to note that Huck et al. (2004) also observe subjects choosing higher output levels than those predicted by the Nash benchmark.

\textsuperscript{37}The Nash benchmarks lie below the relevant lower bounds of the 95\% confidence intervals for average investments, which again reflects the notion that subjects invest a significantly larger amount than prescribed by Nash behavior.
Figure 1: Average investments in the NS-LE treatment with the corresponding 95% confidence intervals, along with the theoretical benchmarks (in panel a, panel b displays NS-HE treatment).
Figure 2: Average investments in the $S\text{-}LE$ treatment with the corresponding 95% confidence intervals, along with the theoretical benchmarks (in panel a, panel b displays $S\text{-}HE$ treatment).
Figure 3: Cumulative distribution of investment choices in asymmetric treatments.

The graph of followers (laggards) in all treatments. This implies that the average investment of leaders exceeds the average investment of followers, and that followers invest more on average than laggards. To substantiate Result 1a, we estimate the following model:

$$\hat{y}_{i,t,k} = \beta_0 + \beta_1 \delta_{\text{leader},k}^i + \beta_2 \delta_{\text{laggard},k}^i + e_{i,t,k}^i,$$

where $e_{i,t,k}^i$ is a residual that is assumed to be independent across groups $k$. For each subject $i$ in group $k$, the preceding equation relates the investment decision in each period $t$ to a constant and two dummy variables that take value 1 if subject $i$’s investment decision is taken in the role of a leader and a laggard, respectively.38

Table 7 gives the parameter estimates for each asymmetric treatment. To illustrate, consider the treatment NS-LE. By construction, the estimate of the

---

38Recall that there are six observations for each subject in a specific asymmetric treatment. As there are five six-player groups and two sessions, we have a total of 360 observations in each of the four asymmetric treatments.
Table 7: Estimation results for the increasing dominance hypothesis.

<table>
<thead>
<tr>
<th>Variable</th>
<th>NS-LE</th>
<th>NS-HE</th>
<th>S-LE</th>
<th>S-HE</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>6.3625***</td>
<td>6.8525***</td>
<td>3.4563***</td>
<td>4.2458***</td>
</tr>
<tr>
<td></td>
<td>(0.1433)</td>
<td>(0.2142)</td>
<td>(0.1345)</td>
<td>(0.2327)</td>
</tr>
<tr>
<td>leader</td>
<td>1.4208***</td>
<td>2.3142***</td>
<td>0.9063***</td>
<td>0.9792***</td>
</tr>
<tr>
<td></td>
<td>(0.1232)</td>
<td>(0.1204)</td>
<td>(0.1585)</td>
<td>(0.2365)</td>
</tr>
<tr>
<td>laggard</td>
<td>−2.8875***</td>
<td>−4.5858***</td>
<td>−1.3792***</td>
<td>−2.3583***</td>
</tr>
<tr>
<td></td>
<td>(0.1508)</td>
<td>(0.2025)</td>
<td>(0.1196)</td>
<td>(0.2202)</td>
</tr>
</tbody>
</table>

Notes: Dependent variable is subject $i$’s period $t$ investment decision in group $k$ ($\hat{y}_{i,t,k}$). 360 observations in each treatment; *** = Significant at the 1% level. Robust standard errors adjusted for clustering on groups in parenthesis.

constant term, which is equal to 6.36 units, reflects the average investment of a follower (see Table 5). The estimated coefficient on the dummy variable $leader$ indicates that, on average, a subject invests 1.42 units more as leader than as follower, amounting to a total average investment of 7.78 units (refer, again, to Table 5). As the estimated coefficient is significantly different from zero ($p$-value < 0.001), we conclude that the average investment of leaders significantly exceeds the average investment of followers. The estimated coefficient on the dummy variable $laggard$ can be interpreted similarly: A subject invests on average 2.89 units less as laggard than as follower. Thus, laggards invest 3.47 units on average, and the difference to the followers’ investment of 6.36 units is significant (at the 1% confidence level).

Similarly, inspection of Table 7 reveals that average experimental investment decisions satisfy the increasing dominance hypothesis in all asymmetric treatments even though subjects do not choose Nash investments, which confirms Hypothesis 1.39

Interestingly, the following result shows that the subjects’ tendency to over-invest relative to the Nash prediction even reinforces increasing dominance.

**Result 1b (Increasing Overinvestment).** The pattern of overinvestment relative to the Nash prediction reinforces increasing dominance as more efficient firms overinvest more than less efficient firms.

The fact that overinvestment tends to increase with the firm-type can be seen most directly in Table 5. To test for significance of the result, we estimate the

39The result also holds when only first-period data are used, so that sequencing effects play no role (the games were presented in a nearly balanced order over the participants in each session). Similar robustness tests were carried out for the results below. Again, we found no evidence for the presence of ordering effects.
Table 8: Estimation results for the increasing overinvestment hypothesis.

<table>
<thead>
<tr>
<th>Variable</th>
<th>NS-LE</th>
<th>NS-HE</th>
<th>S-LE</th>
<th>S-HE</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.623***</td>
<td>0.483*</td>
<td>0.516***</td>
<td>1.036***</td>
</tr>
<tr>
<td></td>
<td>(0.143)</td>
<td>(0.214)</td>
<td>(0.135)</td>
<td>(0.233)</td>
</tr>
<tr>
<td>leader</td>
<td>0.421***</td>
<td>0.314**</td>
<td>0.526***</td>
<td>0.219</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(0.120)</td>
<td>(0.159)</td>
<td>(0.237)</td>
</tr>
<tr>
<td>laggard</td>
<td>−0.688***</td>
<td>−0.186</td>
<td>−0.539***</td>
<td>−0.698**</td>
</tr>
<tr>
<td></td>
<td>(0.151)</td>
<td>(0.203)</td>
<td>(0.120)</td>
<td>(0.220)</td>
</tr>
</tbody>
</table>

Notes: Dependent variable is subject $i$’s period $t$ overinvestment in group $k$ ($\Delta_{i,t,k}^\prime$). 360 observations in each treatment; * = Significant at the 10% level; ** = Significant at the 5% level; *** = Significant at the 1% level. Robust standard errors adjusted for clustering on groups in parenthesis.

$p$-value that types do not matter

0.003 0.057 0.000 0.007

model

$$\Delta_{i,t,k}^\prime = \beta_0 + \beta_1 \delta_{leader,k}^i + \beta_2 \delta_{laggard,k}^i + e_{i,t,k}^\prime.$$ 

Estimates are presented in Table 8. Consider again the treatment NS-LE to illustrate: By construction, the estimate of the constant term, which is equal to 0.62 units, reflects the subjects’ average overinvestment in the role of a follower. As the estimated coefficient on the dummy variable leader is highly significant, the leaders’ average overinvestment of 1.04 units thus exceeds that of followers. Also, the subjects overinvest on average 0.69 units less as laggards than as followers. As this difference in average overinvestment is again significant at the 1% confidence level, Result 1b is confirmed.

Using similar reasoning, Table 8 shows that more efficient firms overinvest more than less efficient firms in treatment $S$-LE (at the 1% confidence level). The estimated differences in the HE-treatments also support Result 1b. Although not all differences are statistically significant, the $p$-values of the hypothesis tests that types do not matter suggest to reject this hypothesis in all treatments at the 10% confidence level, and in three cases at the 1% confidence level. Thus, the tendency towards self-reinforcing dominance is more pronounced than theory would predict.

5.3 The Technological Gap

We now investigate the effect of altering the technological gap between the firms. As detailed in the model section, aggregate investments in the subgame-perfect
Nash equilibrium are predicted to be equal in the symmetric and asymmetric treatments for a given sum of initial efficiency levels (Proposition 2).

Figure 4 therefore compares the corresponding treatments. To illustrate the testing of Hypotheses 2, consider the NS-LE-treatments (Panel a). For each type, the figure gives the difference between its investment in the asymmetric NS-LE-treatment and in the corresponding symmetric treatment. The theoretical prediction is that types 9 and 6.5 invest more in the asymmetric case than in the symmetric case, whereas type 1 invests less. The experimental observations reflect the theoretical prediction not only qualitatively, but also quantitatively. Panels (b) through (d) provide a similar picture. In spite of the substantial effect of asymmetry for the individual types, the right-hand columns in the figures suggest that the higher investments of high types and the lower investments of low types roughly cancel out, as Hypothesis 2 would predict.

To test this, we employ the following model:

$$\hat{y}_{t,k} = \beta_0 + \beta_1 \delta_{sym,k} + e_{t,k},$$

where $e_{t,k}$ is a residual that is assumed to be independent across industries $k$ (but not necessarily across the two replications). For each industry $k$, the preceding equation relates aggregate investments, denoted $\hat{y}_{t,k}$, to a constant and a dummy variable taking value 1 if the observation is generated in a symmetric industry structure.

Table 9 gives the parameter estimates for each comparison. As illustration, we consider the comparison $\Delta^{HE}$ in the no-spillover treatment (i.e., the comparison of the treatments NS-ASYM-HE and NS-SYM-HE). The estimate of the constant term, which is equal to 36.57 units, reflects average aggregate investment at the industry level in the asymmetric industry configuration. The estimated coefficient on the dummy variable $sym$ indicates that average aggregate investment in a symmetric industry configuration is 1.40 units higher, amounting to an average aggregate investment of 37.97 units. As shown in the table, the estimated coefficient is not significantly different from zero at reasonable confidence levels (in the case under consideration, the $p$-value is equal to 0.187). Therefore, there is no statistical evidence suggesting that average aggregate investments are different in the two treatments.

The statistical results are similar in the other comparisons. Thus, individ-

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40 For the S-LE-treatments, theory predicts slightly higher investments in the asymmetric case, whereas average investment in the experiment is slightly higher in the symmetric case. Clearly, however, the deviations are very small in both treatments.

41 Experimental aggregate investments in group $k$ are defined as $\hat{y}_{t,k} = \sum_i \hat{y}_{i,k}$. Our choice of employing aggregate data is motivated by the fact that Hypothesis 2 is about aggregate investments rather than individual investments.

42 Recall that there are two observations for each subject in symmetric treatments and six in an asymmetric ones. As there are five six-player groups and two sessions, we have a total of 80 aggregate observations.

43 Observe that, except for the case $\Delta^{LE}$ in the NS-treatment, average aggregate investments are slightly higher in the symmetric treatments.
Figure 4: Predicted investment change in the asymmetric treatments relative to the corresponding symmetric treatment and the actual average investment change.

uals seem to have well understood incentives to invest: Given two treatments with the same sum of initial efficiency levels, subjects invest less (more) when their efficiency level is lower (higher) in the asymmetric treatment than in the symmetric treatment, as predicted by the theoretical model.

Summing up, we have the following:

Result 2 (Technological Gap). In the asymmetric industry, leaders, followers, and laggards change their investment levels relative to the symmetric industry in the predicted direction in both the low-efficiency and the high-efficiency treatment. Moreover, these changes cancel out at the aggregate level, leaving aggregate investment unaffected.

5.4 Appropriability

Here, we investigate the effects of introducing spillovers on investment behavior. Table 5 shows that average investments of each firm type are considerably lower
Table 9: Estimation results for the comparison of aggregate investments.

<table>
<thead>
<tr>
<th>Variable</th>
<th>No-Spillover Treatments</th>
<th>Spillover Treatments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta^{HE}$</td>
<td>$\Delta^{LE}$</td>
</tr>
<tr>
<td></td>
<td>$\Delta^{HE}$</td>
<td>$\Delta^{LE}$</td>
</tr>
<tr>
<td>$const$</td>
<td>36.5717***</td>
<td>22.7167***</td>
</tr>
<tr>
<td></td>
<td>(0.5269)</td>
<td>(0.6921)</td>
</tr>
<tr>
<td>$sym$</td>
<td>1.4003</td>
<td>0.6833</td>
</tr>
<tr>
<td></td>
<td>(1.0537)</td>
<td>(1.3843)</td>
</tr>
</tbody>
</table>

Notes: $\Delta^{HE}$ and $\Delta^{LE}$ denote the comparison of the corresponding symmetric and asymmetric industry structures. Dependent variable is aggregate investment in industry $k$ in period $t$ ($\hat{y}_{t,k}$). 80 observations in each comparison; *** = Significant at the 1% level. Robust standard errors adjusted for clustering on groups in parenthesis.

in the $S$-treatments than in the $NS$-treatments. We now aim at exploring this finding more thoroughly.

We approach Hypothesis 3 using the following model:

$$\hat{y}_{t,k} = \beta_0 + \beta_1 \delta_{spill,k} + e_{t,k},$$

where $e_{t,k}$ is a residual that is assumed to be independent across groups $k$. This equation relates each subject $i$’s period $t$ investment decision to a constant and a dummy variable that takes value 1 if individual $i$’s observation is assigned to an $S$-treatment.44

Estimating the model, we obtain in the $NS$-treatments, that subjects invest on average 5.99 units (which is the estimate of $\beta_0$). The estimate of $\beta_1$ is equal to -2.41 units with associated $p$-value < 0.001, implying that average investment in the $S$-treatments, which amounts to 3.59 units, is significantly lower. Therefore, Hypothesis 3 is confirmed, which leads to the following result:

**Result 3 (Appropriability).** *Average investments in the spillover treatment are significantly lower than in the no-spillover treatment.*45

Of course, the result does not imply that the efficiency levels of the firms are lower in the spillover treatments than in the no-spillover treatments. To the contrary, ex post efficiency levels are higher in treatments with spillovers than in those without because of the knowledge diffusion effect (see Table 10).46 This

44Recall that there are 16 observations for each subject. As there are five six-player groups and two sessions, the comparison of the two across subjects treatments involves a total of 1,920 observations.

45In fact, the result also holds at the type-level, as a comparison of figures 1 and 2 suggests.

46Ex post efficiency levels are implied by Eq. (2) using average investments presented in Table 5: Add to type $i$’s initial efficiency level the average investment of type-$i$ firms and $\lambda$ times the sum of average investments of the other firm-types in the group.
observation, which is also implied by the Nash equilibrium, is of potential relevance in the debate on the desirability of patent protection: Corresponding to a no-spillover regime, even though patent protection increases investment incentives, it does not lead to higher ex post efficiency levels, at least not in the specific context we are dealing with.

5.5 Deviation from the Cooperative Outcome

We shall now test whether subjects overinvest in the no-spillover treatments and underinvest in the spillover treatments relative to the joint-profit maximizing benchmark, as predicted by Hypothesis 4.

To investigate these claims, we let $y_{i}^{**}$ denote the corresponding JPM benchmark, so that the observed deviation from the JPM prediction can be expressed as

$$\Delta_{i,C}^{t,k} = \hat{y}_{i,k} - y_{i}^{**}.$$

We estimate the model

$$\Delta_{i,C}^{t,k} = \beta_0 + \beta_1 \delta_{spill,k}^i + \beta_2 \delta_{sym,k}^i + e_{i,k}^i,$$

where $e_{i,k}$ is a residual that is assumed to be independent across groups $k$.\(^{47}\)

\(^{47}\)Recall that the dummy variable $\delta_{spill,k}^i$ takes value 1 if the observation belongs to group $k$ in an $S$-treatment (and zero otherwise). Analogously, $\delta_{sym,k}^i$ takes value 1 in symmetric treatments.
Estimation results are presented in Table 6 (Model III). By construction, the estimate of $\beta_0$ gives average overinvestment relative to the cooperative investment decisions in asymmetric NS-treatments. Standard calculations yield that average overinvestment relative to the JPM benchmark amounts to 4.95 units in NS-treatments, which is a highly significant deviation.\footnote{As subjects significantly overinvest relative to the Nash benchmark in the NS-treatments, they do so a fortiori relative to the JPM benchmark. In addition, inspection of Table 5 unambiguously leads to the conclusion that average investments are substantially higher than the relevant JPM benchmarks for all firm-types in the NS-treatments.} Thus, subjects deviate from the JPM benchmark by overinvesting in the presence of negative externalities.

It can be seen from Table 5 that average deviations from the JPM benchmarks are much less pronounced and not unidirectional in the S-treatments. Therefore, a more detailed investigation of the deviations is called for. In symmetric treatments, simple analysis yields that subjects on average underinvest 0.02 units. As this quantity is not statistically different from zero, experimental investment decisions indeed maximize industry profits.\footnote{By construction, the average underinvestment of 0.02 units is the sum of the estimated coefficients presented in Table 6 (Model III). The null hypothesis that $\beta_0 + \beta_1 + \beta_2 = 0$ cannot be rejected; the test’s $p$-value is 0.918.} In asymmetric treatments, however, subjects underinvest relative to the JPM benchmark. In contrast to symmetric treatments, there is significant underinvestment of 0.43 units.\footnote{Here, the test’s null is that $\beta_0 + \beta_1 = 0$. As its $p$-value is 0.003, the hypothesis is soundly rejected.}

Relating to Hypothesis 4, we thus have the following result:

**Result 4 (Deviation from JPM).** Subjects significantly overinvest relative to the JPM benchmark in the no-spillover treatments. In the spillover treatments, subjects approximately choose JPM investment levels in symmetric treatments. In asymmetric treatments, subjects significantly underinvest relative to the JPM benchmarks.

Hence, this result partially supports Hypothesis 4. Clearly, restricting attention to outcomes at the individual level is a very strong test of the theoretical predictions. Surprisingly, however, estimating the model using overinvestment at the group level does not qualitatively affect the findings reported in Result 4.\footnote{The statistical details are available from the authors upon request.}

We therefore have the following observation:

**Observation 3.** From a joint-profit maximizing perspective, aggregate investments are inefficiently high in the no-spillover treatments. In the spillover treatments, in contrast, aggregate investments turn out to be approximately efficient in symmetric treatments and inefficiently low in asymmetric treatments.

To sum up, subjects only manage to coordinate successfully on JPM in symmetric treatments with spillovers. The role of symmetry seems quite clear here: Symmetry provides subjects with a clear common objective, and deviations in
the direction of JPM benefit all subjects in a symmetric way. With asymmetry, coordination is much more difficult. The fact that behavior is more cooperative in the symmetric spillover treatment than in the symmetric no-spillover treatment may result from the framing of the problem as one with positive externalities rather than as a more rivalrous situation with negative externalities (see Andreoni, 1995).

6 Conclusions

Theoretical models explain why markets should be expected to display self-reinforcing dominance under appropriate conditions, but it is hard to identify these mechanisms in real-world markets. We therefore use a laboratory experiment to find out whether subjects’ behavior reflects the crucial strategic effects. We introduce a two-stage investment model which predicts that more efficient firms should invest more into cost reduction than their lagging competitors, thus providing a reason why initial market dominance might be self-reinforcing. It turns out that there is significant overinvestment relative to the Nash benchmark. However, the overinvestment is small, and the increasing dominance hypothesis is confirmed. Moreover, the deviations from the equilibrium follow an interesting pattern. Overinvestment is higher for more efficient types, so that the increasing dominance prediction is reinforced.

Our set-up also allows us to compare aggregate investments in neck-to-neck situations with those in asymmetric leader-laggard structures, confirming the prediction that total investments should be the same in both cases, as long as the average efficiency level is the same in both cases. An interesting extension of our analysis would consider settings where theory predicts differences in both cases, which can happen, for instance, when subjects compete in prices.

Our results also show that spillovers reduce investments in accordance with theory. Finally, the relation between subjects’ decisions and joint-profit maximization is less clear than theory would suggest; in particular, in settings with spillovers, the difference between observed investments and joint-profit maximizing investment levels is insignificant.

Apart from that, however, the conformance between theory and experiments is striking. In spite of the unfamiliar kind of strategic problem, the Nash equilibrium yields surprisingly good predictions. Having confirmed this, it would be interesting to see whether the observed regularities still hold when the product-market stage is modeled explicitly. Deviations in the output stage could have interesting repercussions for investment behavior. Suppose, for instance, that, in the product-market stage, leaders choose higher output levels than in the Cournot equilibrium and, laggards respond by choosing lower outputs. Anticipating this, the leader should set higher outputs than in equilibrium, and the laggard should set lower outputs. Such deviations would reinforce increasing dominance.
Appendix

A.1 Equilibrium under Non-Cooperative Behavior

Firm $i$’s net profit under non-cooperative behavior is given by

$$\Pi^i(y; Y_0, \alpha, \lambda, \kappa) = \left( \frac{\alpha + IY_0^i - \sum_{j \neq i} Y_0^j + (I + \lambda(1 - I))y^i + (2\lambda - 1)\sum_{j \neq i} y^j}{I + 1} \right)^2 - \kappa(y^i)^2. \quad (A.1)$$

In the subsequent analysis, we require that firm $i$’s objective function $\Pi^i$ satisfies the second-order and stability conditions, i.e.

$$\Pi^i_{ii} < 0 \quad \text{and} \quad |\Pi^i_{ii}| > \sum_{j \neq i} |\Pi^i_{ij}|,$$  \quad (A.2)

respectively, where subscripts denote partial derivatives. Letting

$$\alpha^i \equiv \alpha + IY_0^i - \sum_{j \neq i} Y_0^j$$

in (A.1), firm $i$’s first-order condition reads

$$\Pi^i_i(\cdot) = \frac{I + \lambda(1 - I)}{(I + 1)^2} \left( \alpha^i + (I + \lambda(1 - I))y^i + (2\lambda - 1)\sum_{j \neq i} y^j \right) - \kappa y^i = 0.$$

Letting further

$$\beta_1 = \left( \frac{I + \lambda(1 - I)}{I + 1} \right)^2 - \kappa, \quad \beta_2 = \frac{(2\lambda - 1)(I + \lambda(1 - I))}{(I + 1)^2}, \quad \text{and} \quad \beta_3 = \frac{I + \lambda(1 - I)}{(I + 1)^2},$$

firm $i$’s first-order condition may equivalently be rewritten as

$$\beta_1 y^i + \beta_2 \sum_{j \neq i} y^j + \beta_3 \alpha^i = 0. \quad (A.3)$$

Solving for $y^i$ yields

$$R^i(y^{-i}) = -\frac{\beta_3}{\beta_1} \alpha^i - \frac{\beta_2}{\beta_1} \sum_{j \neq i} y^j,$$

which is firm $i$’s best-response function given that rivals invest $y^{-i}$. Noting that $\beta_1 = \Pi^i_{ii}$ and that $\beta_2 = \Pi^i_{ij}$, respectively, we may conveniently rewrite firm $i$’s reaction function as

$$R^i(y^{-i}) = \phi^i - \frac{\Pi^i_{ij}}{\Pi^i_{ii}} \sum_{j \neq i} y^j, \quad \text{with} \quad \phi^i \equiv -\frac{\beta_3}{\beta_1} \alpha^i. \quad (A.4)$$

Thus, using (A.2),

$$\text{sign} \left( \frac{\partial R^i(y^{-i})}{\partial y^j} \right) = \text{sign} \left( \Pi^i_{ij} \right).$$

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As $\text{sign}(\Pi_{ij}) = \text{sign}(\beta_2)$, the fact that
$$\frac{I + \lambda(1 - I)}{(I + 1)^2} > 0; \quad \text{for all } \lambda,$$
implies $\text{sign}(\Pi_{ij}) = \text{sign}(2\lambda - 1)$.

In matrix notation, the system of first-order conditions as given in (A.3) reads
$$\begin{pmatrix}
\beta_1 & \beta_2 & \cdots & \beta_2 \\
\beta_2 & \beta_1 & \cdots & \beta_2 \\
\vdots & \vdots & \ddots & \vdots \\
\beta_2 & \cdots & \beta_2 & \beta_1 \\
\end{pmatrix}
\begin{pmatrix}
y^1 \\
y^2 \\
\vdots \\
y^I \\
\end{pmatrix}
= -\beta_3
\begin{pmatrix}
\alpha^1 \\
\alpha^2 \\
\iddots \\
\alpha^I \\
\end{pmatrix},$$
which we may conveniently rewrite as $M y = -\beta_3 \alpha$. Using the stability condition given in (A.2), which can be restated as $|\beta_1| > (I - 1)|\beta_2|$, the matrix $M$ has a dominant diagonal. Thus, $M$ is known to be nonsingular and $M^{-1}$ exists, whence follows that $y = -M^{-1}\beta_3 \alpha$.

Lemma A.1. The inverse of $M$ is given by
$$M^{-1} = \frac{\beta_2}{(\beta_1 - \beta_2)(\beta_1 - \beta_2 + I \beta_2)}
\begin{bmatrix}
\frac{\beta_1}{\beta_2} + (I - 2) & -1 & \cdots & -1 \\
-1 & \frac{\beta_1}{\beta_2} + (I - 2) & \ddots & \vdots \\
\vdots & \ddots & \ddots & -1 \\
-1 & \cdots & -1 & \frac{\beta_1}{\beta_2} + (I - 2) \\
\end{bmatrix}.$$

Proof. Letting $I$ denote the identity matrix, it suffices to show that $M^{-1}M = I$. For the diagonal elements, we obtain
$$\frac{\beta_2}{(\beta_1 - \beta_2)(\beta_1 - \beta_2 + I \beta_2)} \left( \frac{\beta_1}{\beta_2} + (I - 2) \right) - \beta_2 (I - 1) = 1.$$
Similarly, we obtain for the off-diagonal elements that
$$\frac{\beta_2}{(\beta_1 - \beta_2)(\beta_1 - \beta_2 + I \beta_2)} \left( -\beta_1 + \beta_2 \left( \frac{\beta_1}{\beta_2} + (I - 2) \right) - (I - 2)\beta_2 \right) = 0.$$

Using Lemma A.1, firm $i$'s equilibrium investment can computed to be
$$y^* = -\frac{\beta_2 \beta_3}{(\beta_1 - \beta_2)(\beta_1 - \beta_2 + I \beta_2)} \left( \frac{\beta_1}{\beta_2} + (I - 2) \right) \alpha^i - \sum_{j \neq i} \alpha^j.$$
Simplifying the term in brackets on the right hand side yields
$$\left( \frac{\beta_1}{\beta_2} + (I - 2) \right) \alpha^i - \sum_{j \neq i} \alpha^j
= \left( \frac{\beta_1}{\beta_2} + (I - 1) \right) \left( \alpha + (I + 1) Y_0^i - \sum_{i} Y_0^i \right)
- \left( I\alpha + \sum_{i} Y_0^i \right),$$
so that firm $i$’s equilibrium investment can be rewritten as

$$y^*_i = -\frac{\beta_3 ((\beta_1 - \beta_2)\alpha + (\beta_1 - \beta_2 + I\beta_2) (I + 1) Y_0^i - (\beta_1 + I\beta_2) \sum_i Y_0^i)}{(\beta_1 - \beta_2) (\beta_1 - \beta_2 + I\beta_2)}. \quad (A.5)$$

**Lemma A.2.** Under (A.2), both

$$\beta_1 - \beta_2 < 0 \quad \text{and} \quad \beta_1 - \beta_2 + I\beta_2 < 0.$$ 

*Proof.* By (A.2), $\beta_1 < 0$. If $\beta_2 \geq 0$, we immediately have $\beta_1 - \beta_2 < 0$. If $\beta_2 < 0$, 

$$|\beta_1| > (I - 1)|\beta_2| \iff -\beta_1 > -\beta_2.$$ 

Thus, $\beta_1 - \beta_2 < 0$.

Similarly, if $\beta_2 \leq 0$, (A.2) implies $\beta_1 + (I - 1)\beta_2 < 0$. If $\beta_2 > 0$, we have 

$$|\beta_1| > (I - 1)|\beta_2| \iff -\beta_1 > (I - 1)\beta_2.$$ 

Thus, $\beta_1 + (I - 1)\beta_2 < 0$. 

With Lemma A.2 in mind and noting that $\beta_3 > 0$, $y^*_i > 0$ if and only if the numerator of (A.5) is negative, i.e., 

$$(\beta_1 - \beta_2)\alpha + (\beta_1 - \beta_2 + I\beta_2) (I + 1) Y_0^i - (\beta_1 + I\beta_2) \sum_i Y_0^i < 0,$$ 

or equivalently, if and only if 

$$\alpha > \frac{(\beta_1 + I\beta_2) \sum_{j \neq i} Y_0^j - (I\beta_1 + (I^2 - I - 1)) Y_0^i}{\beta_1 - \beta_2},$$ 

for all $i$. Thus, equilibrium investments are positive if and only if net demand $\alpha$ is sufficiently large relative to the initial efficiency levels $(Y_0^1, \ldots, Y_0^I)$.

**A.2 Proofs of Propositions 1 through 4**

*Proof of Proposition 1.* From (A.5), it follows that 

$$y^i - y^j = \frac{\beta_3 (I + 1)}{\beta_1 - \beta_2} \left( Y_0^i - Y_0^j \right).$$ 

As $\beta_3 > 0$, the claim follows from Lemma A.2. This completes the proof.

*Proof of Proposition 2.* (i) Let $y \equiv \sum_i y^i$ denote aggregate investment. From (A.5), we obtain 

$$y = \frac{\beta_3 (I\alpha + \sum_i Y_0^i)}{\beta_1 - \beta_2 + I\beta_2}.$$ 

Hence, the numerator is determined by $\sum_i Y_0^i$ only. This completes the proof. (ii) follows immediately from Proposition 1 and part (i) of this proposition.

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52Note that in the case of symmetric firms with initial efficiency level $Y \geq 0$, the restriction on $\alpha$ boils down to $\alpha + Y > 0$, or equivalently, to $a > c - Y$. 

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Proof of Proposition 3. From the discussion in Footnote 15, player $i$’s optimal investment $y^i(\lambda = 0.5)$ is independent of rivals’ actions. Note that condition (6) implies

$$\frac{\partial \Pi(\lambda < 0.5)}{\partial y^i} > \frac{\partial \Pi(\lambda = 0.5)}{\partial y^i} > \frac{\partial \Pi(\lambda > 0.5)}{\partial y^i}.$$ 

Hence, for arbitrary $\lambda < 0.5$ and arbitrary actions of the competitors, the best response is above $y^i(\lambda = 0.5)$, whereas for arbitrary $\lambda > 0.5$, the best response is below $y^i(\lambda = 0.5)$. This completes the proof.

Proof of Proposition 4. We prove the statement for $\lambda \in (0, 1]$. In a Nash equilibrium, $\frac{\partial \Pi(y^*)}{\partial y^i} = 0$. Therefore,

$$\frac{\partial \Pi(y^*)}{\partial y^i} = \frac{\partial \Pi'(y^*)}{\partial y^i} + \sum_{j \neq i} \frac{\partial \Pi'(y^*)}{\partial y^j} = (I - 1)(2\lambda - 1) > 0,$$

where the last two steps follow from Observation 1. As $\frac{\partial \Pi(y^*)}{\partial y^i} > 0$, for all $i$, concavity implies $\frac{\partial \Pi(y)}{\partial y^i} > 0$, for all $y < y^*$. Thus no $y \leq y^*$ can maximize the firms’ joint profit, whence follows for all $i$ that $y^{**}(\lambda) > y^i(\lambda)$. The proof for the case $\lambda \in [0, 0.5)$ is analogous, and therefore omitted. This completes the proof.

References


Budd, Christopher, Christopher Harris, and John Vickers (1993) ‘A model of the evolution of duopoly: Does the asymmetry between firm tend to increase of decrease?’ Review of Economic Studies 60(204), 543–573.


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