What Can European Sports Leagues Learn from the Major Leagues

Helmut Dietl, Egon Franck and Tariq Hasan

June 2005
What Can European Sports Leagues Learn From The Major Leagues?
Why Professional Sports Leagues Should Be Organized As Cooperatives

Helmut Dietl
University of Zurich
Institute for Strategy and Business Economics
helmut.dietl@isu.unizh.ch

Egon Franck
University of Zurich
Institute for Strategy and Business Economics
egon.franck@isu.unizh.ch

Tariq Hasan
University of Zurich
Institute for Strategy and Business Economics
tariq.hasan@isu.unizh.ch

Preliminary Version; June 1, 2005.

Abstract

Professional sports leagues in Europe and the United States exhibit many differences. Among others such as the existence of mechanisms providing disincentives for spending (e.g. salary caps) the fundamental difference is the organizational arrangement of clubs and the governing body of the league. The U.S. Major Leagues are organized in a manner similar to cooperatives in which team owners make decisions by majority voting. In Europe however the leagues themselves are legally and economically independent entities which buy team-output from the clubs. We claim that due to the specificity of investments in sports clubs any governance between teams and the league that occurs via the marketplace will lead to inefficiencies partly because clubs will use resources to protect their investments. This paper consists of a theoretical comparative institutional analysis. The model derived in this paper shows that a change from a market-governed organizational regime to a cooperative is accompanied by an increase in welfare. Thus, there is something European sports leagues could learn from their American counterparts.

JEL-Classification: L83, L14, D23
1 Introduction

Production of spectator sports occurs on two stages: Firstly, there is the level of production of single teams, where club-owners invest money into player talent, training facilities and assistance of all sorts. The teams themselves then act as inputs on the second level of production, on which the championship-race is produced. At the institutional level these stages of production can be more or less vertically integrated.

In the case of extreme integration a league firm with teams as subsidiaries would emerge. It is quite obvious why this alternative has not been able to serve as a successful role model in practice so far. On the one hand this organizational structure is in conflict with the requirement of securing the integrity of the championship race. An integrated league functions under unified ownership. The league-owner would be capable of and, at the same time, suspected of influencing or “making” rules at his discretion in order to alter the behavior of his subsidiaries according to his own strategy. Therefore, the classical firm is handicapped by an additional cost of securing and credibly signaling that the owner keeps out of the policy of the subsidiaries. Presumably, this is rather difficult to achieve since it needs to be done “against” the institution of ownership, which by its very nature includes the right to intervene, e.g. arrange the outcome of sporting competition in this case. Moreover, and arguably more important, the firm structure induces a moral hazard problem. Team owners will be replaced by employed managers as clubs become subsidiaries of a unified firm. It seems reasonable to assume that the effort managers exert in team-development is not observable by the central league authority. This may be a consequence of the fact that the local markets of the subsidiaries differ greatly due to historical, cultural or ethnical peculiarities. In this case local and implicit knowledge becomes important for making value-enhancing decisions at the club level. Such knowledge cannot be effectively monitored by a central league owner. Moreover, the league owner cannot infer managerial effort by observing output, for example by looking at the championship performance of a team. There is no method to find out if a certain rank in the championship is the result of little managerial effort and many lucky circumstances or vice versa. Since managerial effort is not contractible as a consequence, the firm-solution comes at the price of a nontrivial problem to provide efficient managerial incentives.

In practice two basic “less integrated” alternatives of league organization can be studied. Within the spectrum of organizational forms between markets and hierarchies (Williamson (1975)) the U.S. Major Leagues (MLB, NHL, NFL, NBA), which are organized in a manner similar to cooperatives, can be termed as genuine hybrids. All issues of league-wide concern are decided by the owners of the member-teams through mechanisms of majority voting. These team-owners are the central source of power in the U.S. Major Leagues. They are autonomous entrepreneurs when it comes to managing their teams and at the same time voting members of a cooperative association that is taking care of championship production. Consequently, the U.S. structure of league organi-
zation emerges through a partial vertical forward-integration of all team-owners into championship production.

There is no such form of forward-integration of the teams into league organization in European sports leagues. The governing body of the league and the teams are either completely autonomous agents like for example in Formula One Motor Racing. Or, as it is the case in most national soccer leagues, they are only loosely coupled through membership in the all-encompassing national association governing the entire sport. European clubs do not (jointly) own the league organization responsible for championship management as their North American counterparts do. European clubs are therefore not the central source of power in league organization. Within the spectrum of organizational forms between markets and hierarchies, the European relationship between the teams playing in the championship and the league governing body is therefore much closer to a market-interface. For the sake of simplicity we will use the term market in this paper and contrast it to the cooperative alternative typical for the U.S.

The question discussed in this paper is straightforward: Which of the two organizational forms found in practice – the European market alternative or the American cooperative – is superior? We will answer this question based on a theoretical comparative institutional analysis. The model derived in this paper will show that welfare, as measured by aggregate profits of all agents involved, increases when changing from a market-governed organizational regime to a cooperative. Obviously, there is something European sports leagues could learn from their American counterparts.

A vast amount of research has been devoted to the peculiar differences between U.S. and European sports leagues. Rosen and Sanderson (2000) compare the two systems regarding labor market effects. Noll (2000) discusses efficiency effects both of promotion and relegation vs. closed leagues as well as effects of staging a post-season tournament as it is done in North America. Some researchers (see e.g. Fort (2000)) conclude that the similarities between the two ‘types’ of spectator sports outweigh the differences and are somewhat perceived rather than factual. However large the literature, not much research has been conducted concerning the organizational differences governing professional sports on the two continents. As Szymanski (2003) points out, ‘extending the analysis of team sports to assess the effect of the strikingly different institutions of soccer offers a rich laboratory for researchers’. We try to step into this laboratory with this paper.

The rest of the paper is organized as follows. Section two gives a brief overview of the economic peculiarities of sports production. In the following section we will present a contest-model with an endogenously determined outside opportunity. Subsequently we will show that whenever the outside opportunity is sufficiently profitable, inefficient rent-seeking on behalf of some subset of clubs will occur. However, even in the absence of such investments, a cooperative is favorable in terms of aggregate profits. Section four discusses the results and provides some hints for future research.
2 Some Peculiarities of Sports Production

In order to understand the economic intuitions that frame the design of league structures it is useful to briefly review some economic peculiarities of the production of spectator sports. The concept of a ‘championship’ possesses an important implication stemming from the fact that a ‘champion’ is to be determined. The validity of the ‘championship’ mainly rests on its monopoly status. If there are several championships per one market area and sports, no consistent ranking of all performers is achieved and hence, the championship will lose a significant part of its value for consumers. A brief look at the history of major league sports shows that the periods of inter-league competition have been rather short and ended in mergers if the contender succeeded in seriously challenging the established league at all.\(^1\) In European soccer this uniqueness of a national championship is additionally enforced on a formal basis by the European Football Association (UEFA) via lack of approval for any national league not administered by the respective national soccer association.

Due to the often definitory monopoly status of major leagues, investments of club-owners into the their teams are specific in the sense that they cannot be transferred to alternative, equally profitable endeavours. Any individual club-owner has no economically viable exit-option from a monopolistic major league other than shutting down and selling the teams. Whenever teams and the league coordinate their relations via the market, a hold-up risk (Williamson (1975)) arises. Having made investments into the teams, club-owners do not possess any outside-opportunity and hence are forced to accept whichever conditions are offered by the league body. While in European soccer the magnitude of such issues is dampened by the fact that the league bodies are administered to some extent by the national sports associations who do not act as pure profit maximizers, the full extent of such a situation is felt in Formula One Motor Racing. While any single club-owner cannot produce a championship race alone, some subset of clubs may be tempted to threaten to set up some competing league - even though the probability of success of such a league might be low a priori. This is exactly what is happening in Formula One Motor Racing, where a majority of racing teams threaten to cancel the ‘concord agreement’, the agreement governing relations between the team association (FOCA) and F1 Management, in order to start an own racing league dubbed Grand Prix World Championship (GPWC). Similar behavior, albeit somewhat more defensive, can be observed in European soccer, where the originally 14 and presently 18 most powerful European soccer clubs formed the ’Group of 14’ (G14) in order to augment their bargaining power versus the respective national leagues and the UEFA.

These endeavours of soccer clubs and F1 racing teams are essentially investments into outside options in order to augment their bargaining power versus the league and are therefore merely instruments used to affect the distribution of rents. A standard remedy in the presence of specific investments that helps

\(^1\)See Qurik and Fort (1992, pp. 294-361) for a review of such events.
avoiding unproductive rent-seeking is vertical integration of the two levels of production (Klein, Crawford & Alchian (1978), Williamson (1975)).

It has already been argued in the previous section that while the unification of club owners and the league body under one single corporate 'roof' solves the hold-up problem, it implies new issues (e.g. problem of integrity, moral hazard) which we believe are even more costly in terms of welfare. An organization as a cooperative possesses major advantages over the 'corporate organization'. Clubs remain economically and legally independent from which it follows that incentives for club-owners are not distorted. Additionally, since club-owners can influence matters affecting the league as a whole, incentives to exit the league are less if not inexistent. The model derived in the following section will show that welfare, as measured by aggregate profits of all players, increases when changing from a market-governed organizational regime to a cooperative. This is due to the absence of the independent, profit-maximizing league body and the lack of investments into the outside-opportunity.

3 Model Setup

As has been argued in the previous section, the inherent monopoly status of any championship race renders investments into the team product, i.e. the championship, specific by nature. Having made investments into the team, such as players, support staff, infrastructure and so forth, club-owners do not have any viable alternative to playing in the league other than shutting down and selling the team. When relations between clubs and the league body are governed by the market, this specificity of investments leads to a high degree of vulnerability on the side of the clubs. Any league seeking to maximize profits will lower the transfers accruing to the teams out of championship play. However, anticipating this behavior, club-owners are able to make up-front investments into a generic outside-opportunity signaling that they are willing to exit the league which is exactly what is observed both in Formula 1 motor racing and in the major European soccer leagues. These threats must be taken seriously by any league organizer. Even though investments into the team are of specific nature, the league is fully depending on teams participating in league play. Therefore, the threat of exiting the league may serve as an instrument to appropriate rents on behalf of some subset of teams.

In this section we develop a model showing how a cooperative organization of a professional sports league produces a favorable outcome when compared to a situation in which actions between teams and the league are coordinated via the marketplace. We will do so by combining a rather standard contest model with the possibility of augmenting the value of some generic outside opportunity - e.g. the endeavours regarding the GPWC in motor racing.

Suppose there are two teams \( i = 1, 2 \) which can either engage in a championship administered by the league body or choose to pursue an outside opportunity the value of which is determined endogenously. The championship is modeled as a standard contest along the lines of Tullock (1980). That is, contingent
on joining the league, the clubs compete for the league prize $v$. The probabilities of success are supposed to be non-discriminating logit contest-success-functions, i.e. $p_i = \frac{e_i}{e_1 + e_2}$, where $e_i$ denotes club $i$'s effort when engaging in league play. Note that effort is not merely player effort on the pitch but rather encompasses all investments into the team which augment the probability of success such as player talent, medical assistance, infrastructure and so forth. Since it is assumed that the prize will be won by one of the two clubs with certainty, it must be the case that $p_1 = 1 - p_2$. For reasons of simplicity, effort costs are assumed to be linear resulting in constant marginal costs of effort. Asymmetry is incorporated in the model via effort costs. Club 1 possesses an advantage over club 2 in the sense that it is able to produce any given level of effort at a lower cost. Total league revenue $R(e_1, e_2)$ is assumed to be a concave function of aggregate effort. This reflects the fact that demand for league games increases with increased quality of play which again is incorporated in effort.\footnote{We have neglected the possibility of demand being affected by competitive balance. Empirical findings for or against the relevance of competitive balance have been highly mixed, see Szymanski (2003) for a summary. It remains to be shown that the results derived below hold true in a setting in which revenue is not independent of competitive balance.} Throughout this paper, we will assume that total revenue is given by $R(e_1, e_2) = (e_1 + e_2)^\frac{1}{2}$. Whether the derived results can be generalized to all concave revenue functions is subject to future research.

Prior to joining the league, both teams may invest some amount $z_i$ into the outside opportunity, the value of which is given by $a(z_i) = r z_i^0.5$, where $r \in (0, 1)$ in order to ensure that league production is ex ante desirable from a social point of view.\footnote{Note that the derived are not sensitive to the functional form of the outside option.} These investments serve to increase the credibility of the threat of league-exit and include measures such as founding the 'group of 14' and providing the group with a corporate identity and so forth. It is important to note that the costs of these investments into the outside opportunity are sunk.

In the next subsection we will analyze how teams and the league behave in a setting in which relations between the individuals are governed by the market. In the subsequent subsection a situation in which clubs form a cooperative similar to U.S. Major Leagues is analyzed. Then, the results will be compared and discussed.

3.1 Market Interaction

In this setting we will look at a situation in which there are two clubs and a profit maximizing league body. The league bodies of the major European soccer leagues such as the German Football League (DFL) are not commonly perceived as profit maximizers since they are in part governed by the national soccer associations and, more importantly, pursue some secondary goals such as promoting the sport as a whole in society. Nonetheless we think that the assumption of profit maximization on behalf of the league is not very far-fetched since secondary goals can be met to a larger extent if profits are higher. Further
thermore, there also exist leagues which are 'pure' profit-maximizers; Formula 1 motor racing, where the issues addressed in this paper are most acute, as one example.

When choosing to join the league and participating in the contest staged by the league body, club-owners exert effort \( e_i \) in order to compete for the championship prize \( v \). However, the league body may award some share \( 1 - k \leq \frac{1}{2} \) of total prize money to the team finishing second. Then, given that the teams participate in the league, expected profits are given by:

\[
E(\pi_1) = p_1kv + p_2(1-k)v - \beta c_1 e_1 - c_2 z_1 \\
= \frac{(e_1 - e_2)k + e_2}{e_1 + e_2}v - \beta c_1 e_1 - c_2 z_1 \\
E(\pi_2) = p_2kv + p_1(1-k)v - c_1 e_2 - c_2 z_2 \\
= \frac{(e_2 - e_1)k + e_1}{e_1 + e_2}v - c_1 e_2 - c_2 z_2
\]

where \( \beta \in (\frac{1}{4}, 1) \) represents club 1’s effort cost advantage. The outside opportunity and league participation are mutually exclusive alternatives for the clubs. Thus, the expected profits above are profits contingent on league participation.

The league is providing the teams with the organization of the championship as a whole. That is, the league is administering the rules of play, scheduling games and so forth. It is assumed that the league body is the holder of the residual right which implies that the league passes the prize \( v \) to the clubs and is able to keep the residual revenue. As mentioned above, total revenue out of championship play is given by \( R(e_1, e_2) = (e_1 + e_2)^2 \). Facing investments into the outside opportunity on behalf of the teams, the league body will thus maximize \( R(e_1, e_2) - v \) taking into account that the prize \( v \) and the share awarded to the winner \( k \) has to be such that both teams prefer participating in the league to going into their outside opportunities. In other words, in equilibrium \( v \) and \( k \) have to be chosen in a manner satisfying individual rationality constraints on behalf of both teams. This is due to the lack of alternative income sources for the league, which implies that club-participation is the only possibility for the league to generate positive profits.

Since we have restricted the parameter determining outside-opportunity profitability to be less than unity we can without loss of generality assume that \( c_2 \equiv 1 \). Then, the parameter \( c_1 \) can be interpreted as a measure of relative effort costs. However, it will henceforth be assumed that \( c_1 \in (0, 1] \).

The timing of events is as follows:

1. Teams select their outside opportunity investment levels \( z_i, i = 1, 2 \).
2. Observing \( z_i \) the league makes a take-it-or-leave-it offer \((v, k)\) to the teams.
3. Teams choose whether to accept the offer and participate in the championship with effort levels \( e_i \) or go into their respective outside opportunities.
4. Payoffs are realized.
The model will be solved using backward induction. Thus, when analyzing the behavior of club-owners in stage 3, we will assume that teams have decided to participate in the league after the league’s offer \((k, v)\).

Having already decided to participate in the league and to compete for the league prize \(v\), effort-choices of the club-owners will be such to maximize their respective expected profits. Thus, team \(i\) solves

\[
\max_{e_i} E(\pi_i) \tag{3}
\]

where \(E(\pi_i)\) is given above by equations (1) and (2) respectively. Then, the FOC are given by

\[
e_2(2k - 1) = \beta c_1 \tag{4}
\]

\[
e_1(2k - 1) = c_1 \tag{5}
\]

where \(\hat{e}_i = \arg\max_{e_i} E(\pi_i)\) for \(i = 1, 2\). Solving this system of reaction functions for the respective equilibrium effort levels of the subgame beginning in stage 3 yields

\[
(\hat{e}_1(v, k), \hat{e}_2(v, k)) = \left(\frac{v(2k - 1)}{c_1(1 + \beta)^2}, \frac{\beta v(2k - 1)}{c_1(1 + \beta)^2}\right) \tag{6}
\]

In line with standard contest results, equilibrium effort levels increase with the spread between first and second prize.\(^4\) Additionally, increasing effort costs lead to less effort in equilibrium. Increasing team-heterogeneity (i.e. a lower \(\beta\)) leads to more effort of the more productive club (club 1) and less effort of the less productive club (club 2) in equilibrium. As asymmetry has been incorporated via marginal costs, a higher degree of asymmetry is equivalent to lower marginal costs of club 1 which thus will increase effort up to the point in which marginal costs and marginal revenue are equal. The contrary holds true for club 2. As a reaction to the increased effort level of club 1, club 2 will lower its own effort. This is an effect resulting from the strategic complementarity of efforts.

Anticipating the behavior of club-owners in stage 3, the league body will select the championship prize and distribution among participants so as to maximize its profits in stage 2. As has been shown, the measure which matters for equilibrium effort is the spread between first and second prize \(v(2k - 1)\). Therefore, when acting unconstrainedly, the league will - in order to maximize revenue - maximize the spread. Two points are worth noting. Firstly, the league is able to affect the spread between first and second prize both via \(k\) and \(v\). While an increase in \(v\) will lower the profits of the league as the holder of the residual right, changes in \(k\) will not alter league profits. Therefore, if possible, the league will set \(k = 1\). Secondly, the league cannot act in an unconstrained manner. As noted above, the league has to ensure participation on behalf of both of the clubs since they will choose their outside opportunity if league-participation is

\(^4\)The spread between first and second prize is given by \(kv - (1 - k)v = (2k - 1)v\).
less profitable. As a consequence, the league has to choose a vector \((k,v)\) to ensure that both of the clubs are at least as well off as in their respective outside opportunities. As is shown in the following, when dealing with individual rationality constraints, in equilibrium the league can entirely focus on club 2 since club 1’s IR-constraint is satisfied whenever club 2’s is. To see this, first note that given the equilibrium effort levels \((\hat{e}_1, \hat{e}_2)\) of the contest subgame, the probabilities of success reduce to

\[
(p_1(\hat{e}_1, \hat{e}_2)) = (p_1, p_2) = \left( \frac{1}{1+\beta}, \frac{\beta}{1+\beta} \right) \quad (7)
\]

Winning probabilities are a function of club-heterogeneity only. In the absence of investments in outside opportunities, expected profits are given by

\[
E(\pi_1 | z_1 = 0) = \frac{1}{1+\beta}kv + \frac{\beta}{1+\beta}(1-k)v - \beta \frac{v(2k-1)}{(1+\beta)^2} \quad (8)
\]

\[
E(\pi_2 | z_2 = 0) = \frac{\beta}{1+\beta}kv + \frac{1}{1+\beta}(1-k)v - \beta \frac{v(2k-1)}{(1+\beta)^2} \quad (9)
\]

Note that \(E(\pi_1 | z_1 = 0) \geq E(\pi_2 | z_2 = 0)\). Since both clubs are equally productive in their outside opportunities, they will - in equilibrium - invest equal amounts in their outside opportunities. Thus, it will be the case that in equilibrium \(z_1 = z_2\) and consequently \(E(\pi_1) \geq E(\pi_2)\). This again implies that in equilibrium only club 2’s IR-constraint will be binding, i.e. the league will choose a vector \((v,k)\) such that club 2 prefers joining the league.\(^5\) This will also ensure that team 1 prefers participating in the league.

The league’s problem thus is the following

\[
\max_{v,k} \{ R(\hat{e}_1, \hat{e}_2) - v \} = \max_{v,k} \{ (\hat{e}_1 + \hat{e}_2)^{1/2} - v \} \quad (10)
\]

\[
s.t. \quad E(\pi_2) \geq rz_2^{0.5} - z_2 \quad \quad k \in [\frac{1}{2}, 1]
\]

where the left-hand side of the IR-constraint is given by (9) minus the costs of investing in the outside opportunity \(z_2\). The solution to this problem is summarized in the following proposition:

**Proposition 1** Suppose that \(\beta \in (\frac{1}{2}, 1)\) and \(c_1 \in (0,1]\). Then, the solution \((\hat{k}, \hat{v})\) of problem (10) is given by

\[
\hat{k} = \begin{cases} 
\frac{1+3\beta+2\beta^2+4c_1r_2^{0.5}(\beta^2-2\beta-1)^2}{(1+2\beta-\beta^2)[1+\beta-8c_1r_2^{0.5}(\beta^2-2\beta-1)]} & \text{if } rz_2^{0.5} > a_1 \\
1 & \text{elsewhere}
\end{cases}
\]

It is henceforth assumed that in case of indifference between outside opportunity and league play, club 2 will prefer engaging in the league championship.
\[
\hat{v} = \begin{cases} 
2r z_2^{0.5} + \frac{1+\beta}{4c_1(1+2\beta-\beta^2)} & \text{if } r z_2^{0.5} > a_1 \\
\frac{r z_2^{0.5}}{\beta^2} & \text{if } a_0 \leq r z_2^{0.5} \leq a_1 \\
\frac{1}{4c_1(1+\beta)} & \text{elsewhere}
\end{cases}
\]

where
\[
a_0 = \frac{\beta^2}{4c_1(1+\beta)^3}
\]

and
\[
a_1 = \frac{\beta^2(1+\beta)}{4c_1(\beta^2-2\beta-1)^2} > a_0
\]

Proof. See appendix A.1

The profit-maximizing prize \( \hat{v} \) offered by the league is a non-decreasing function of investments into the outside option by club 2 and the parameter \( r \) determining the value of the outside option, i.e. \( \hat{v} = f(a(z_2)) \equiv \hat{v}(z_2) \). Analogously, \( \hat{k}(z_2) \) is a non-increasing function of the value of club 2’s outside opportunity. Note that both \( \hat{v}(z_2) \) and \( \hat{k}(z_2) \) are continuous. The two functions \( \hat{v}(z_2) \) are plotted in figures 1 and 2 below for some given set of parameters \( r, c_1 \).

![Figure 1: The league’s profit maximizing prize as a function of the value of club 2’s outside opportunity.](image)

Interestingly, as long as \( rz_2^{0.5} < a_0 \) the IR-constraint of club 2 is not binding. This is also represented in the above figures since for \( rz_2^{0.5} < a_0 \) on the one hand we have \( k = 1 \) and on the other hand, the optimal prize \( \hat{v} \) is constant. Thus, facing very low values of the outside opportunity, the league can attain its global profit maximum given the subsequent contest and the constraint on \( k \). The reason for this is quite simple: In order to maximize its profits, the league has to generate revenue for which it has to ensure that the participating clubs
exert some level of effort. This again implies that the league has to pass some amount of total revenue back to the clubs, leaving them with higher profits than they would attain under the rather unprofitable outside opportunity. In sports in which the outside opportunity is rather unprofitable - whether this is due to high costs or low productivity - the league knows that the clubs will not be willing to exit the league. Consequently, the league can act as if it were unconstrained. As club 2’s investment in the outside opportunity and/or the profitability of the outside opportunity and subsequently its value increases, the latter will pass a threshold after which club 2 prefers not to join the league were the league not to increase expected profits of club 2. The league can either do so by increasing the prize or - since it is more probable that club 2 finishes the league second than first
- decrease the share of the prize awarded to the champion. As stated above, in order to maximize profits, the league will try to keep the contest a ‘winner-takes-all’ contest as long as possible. This is also reflected in the two figures above. The league will first increase prize money and keep the contest a ‘winner-takes-all’-contest. Only once the value of club 2’s outside opportunity passes a second threshold, i.e. \( r z_2^{0.5} > a_1 \), the league will reduce the fraction of the prize awarded to the winner while still increasing prize money albeit at a lower rate. This happens when marginal costs of increasing the prize money \( v \) surpass marginal costs of reducing the share awarded to the winner \( k \) via reduced efforts.

As can be seen, in the equilibrium of the contest subgame, in the presence of a productive outside opportunity of club 2, the league will increase the prize money \( \hat{v} \) with increasing investments \( z_2 \) into the outside opportunity. This illustrates how investments into the outside opportunity can act as a means of rent appropriation. Since club 2 knows that a functioning championship race

Figure 2: The league’s profit-maximizing prize-sharing parameter as a function of the value of club 2’s outside opportunity.
is impossible without its participation, it will use this knowledge to increase its share of total surplus. Thus, even though the formal bargaining power lies in the hand of the league - it is able to make some take-it-or-leave-it offer \((v, k)\) to the clubs - the factual bargaining power rests with the less productive of the two clubs which can determine the outcome via its investments in the outside opportunity. However, the threat of league exit and the subsequent appropriation of rents by the ‘weaker’ of the two clubs can only occur if the outside opportunity is sufficiently profitable and if investments into the outside opportunity are sufficiently high. This helps understand why European soccer clubs not only founded the group of 14 but also institutionalized the group by providing it with a headquarters, annual meetings and so forth. Merely founding the group implied an investment level too low to credibly exert the bargaining power. Analogously, the negotiations of GPWC-racing teams with track-owners and race promoters can be viewed as investments augmenting the value of the outside opportunity of race teams, i.e. their own racing league.\(^6\)

Having outlined the profit-maximizing behavior on behalf of the league we will now proceed to stage 1, in which the two clubs choose the amount to be invested into the respective outside opportunities. The following proposition summarizes stage 1-investment behavior of club 2:

**Proposition 2** Suppose that \(\beta \in (\frac{1}{4}, 1), r \in (0, 1)\) and \(c_1 \in (0, 1]\). Then, facing \((\hat{k}, \hat{v})\) as derived in proposition 1, club 2 will always join the league. Positive investments in the outside opportunity by club 2 will be made if the outside opportunity is relatively profitable to league interaction, that is, if

\[
a(z_2^*) - z_2^* \geq E(\pi_2 | z_2 = 0)
\]

\[\iff r^2 \frac{1}{2} - \left(\frac{r}{2}\right)^2 = r^2 \frac{1}{4} \geq \frac{\beta^2}{4c_1(1 + \beta)^3}\]  \(\text{(11)}\)

where \(z_2^* = \arg\max_{z_2} \{a(z_2) - z_2\}\).

**Proof.** See Appendix A.2 \(\blacksquare\)

Thus, for club 2 to make any investments into the outside opportunity it must be the case that the outside opportunity is relatively profitable. The reason for this stems directly from the derivation of proposition 1. The crucial point is that there exists \(\hat{v} = \frac{1}{4c_1(1 + \beta)}\) which globally maximizes league profits. If club 2’s outside opportunity is rather unproductive (i.e. condition (11) is violated) then it will be the case that club 2 is better off joining the league at \(v = \hat{v}\) and \(k = 1\) than at the profit maximum of the outside opportunity - which

\(^6\)Interestingly, the most successful team of past years, Scuderia Ferrari, has not joined the GPWC but has signed the new ‘concord’ agreement with F1 management which is in line with the finding that it is the weaker teams which threaten the league body with league-exit.
is the profit accruing to club 2 once its IR-constraint is binding. Thus, in such a scenario club 2 will restrain from investing into the outside opportunity and be happy to join the league without bearing any investment costs. However, as the outside opportunity becomes more productive or league interaction less attractive due to higher costs $c_1$, the league will have to deviate from its globally desired prize $v = \hat{v}$ and ensure club 2’s participation by increasing prize money $v$ and - at a later stage - also increasing the share of the prize awarded to the loser.\footnote{Note that this result holds for all functional forms of the outside opportunity as long as the outside opportunity is relatively unproductive in the sense that the league is always able to make club 2 indifferent and still enjoy nonnegative profits. The latter is ensured by $r^2 \leq \frac{1}{4c_1(1+\beta)}$. If this were not the case, then the outside opportunity then the league could not afford to ensure club 2’s participation. We will discard this case by assuming that this condition is satisfied due to the fact that in such circumstances it is not efficient to play in a league in the first place.}

The result derived in proposition 2 shows that inefficient rent-appropriation measures on behalf of some subset of the teams is an issue only if the outside opportunity of the teams is relatively profitable. Thus profit-maximizing league bodies in sports in which there is a market for but one league - and consequently a competing league founded by exiting teams were relatively unprofitable - need not fear any ‘rioting’ behavior on behalf of the clubs. Note also that as long as the league can afford to pay off the clubs threatening to exit the league it will do so.

Summarizing the results derived in propositions 1 and 2, the prize structure, investment- and effort-levels that prevail in equilibrium are the following:

\begin{align*}
    z_1^* &= 0 \\
    z_2^* &= \begin{cases} 
        0 & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)} \\
        \argmax_{z_2} 0.5 - z_2 = \frac{r^2}{4} & \text{otherwise}
    \end{cases} \\
    k^* &= \begin{cases} 
        1 & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)} \\
        \frac{1+3\beta+2\beta^2+4c_1r_2^{0.5}(\beta^2-2\beta-1)^2}{(1+\beta-\beta^2)(1+\beta-8c_1r_2^{0.5}(\beta^2-2\beta-1))} & \text{otherwise}
    \end{cases} \\
    v^* &= \begin{cases} 
        \frac{1}{4c_1(1+\beta)} + \frac{1}{1+2\beta-\beta^2} & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)} \\
        2r_2^{0.5} & \text{otherwise}
    \end{cases} \\
    \hat{e}_1(v^*, k^*) &= \begin{cases} 
        \frac{1}{4c_1^2(1+\beta)} & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)} \\
        \frac{\beta}{4c_1^2(\beta^2-2\beta-1)^2} & \text{otherwise}
    \end{cases} \\
    \hat{e}_2(v^*, k^*) &= \begin{cases} 
        \frac{1}{4c_1^2(1+\beta)} & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)} \\
        \frac{\beta}{4c_1^2(\beta^2-2\beta-1)^2} & \text{otherwise}
    \end{cases}
\end{align*}
Plugging the respective effort- and investment levels as well as the equilibrium prize structure into expected profits yields the following (expected) profits:

\[
E(\pi_1) = \begin{cases} 
\frac{1}{4c_1(1+\beta)^2} + \frac{(1-\beta)(1+\beta)^2}{4c_1(\beta^2-2\beta-1)^2}, & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)^2} \\
\frac{1}{2}r^2, & \text{otherwise}
\end{cases} \tag{18}
\]

\[
E(\pi_2) = \begin{cases} 
\frac{1}{4c_1(1+\beta)^2} - \frac{\beta^2}{c_1(1+\beta)^2}, & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)^2} \\
\frac{1}{4c_1(1+2\beta^2)^2}, & \text{otherwise}
\end{cases} \tag{19}
\]

League profit is given by

\[
\pi_L = R(\hat{e}_1, \hat{e}_2) - v = \begin{cases} 
\frac{1}{4c_1(1+\beta)} - \frac{\beta^2}{c_1(1+\beta)^2}, & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)^2} \\
r^2, & \text{otherwise}
\end{cases} \tag{20}
\]

Then, total welfare equals aggregate profits and amounts to

\[
E(W_M) = \begin{cases} 
\frac{\beta^2 + \beta + 1}{4c_1(1+\beta)^2} + \frac{1+\beta}{4c_1(1+2\beta^2)^2} - \frac{r^2}{4}, & \text{if } r^2 < \frac{\beta^2}{c_1(1+\beta)^2} \\
\frac{(1-\beta)(1+\beta)^2}{4c_1(\beta^2-2\beta-1)^2} + \frac{(1+\beta)^2}{4c_1(\beta^2-2\beta-1)^2}, & \text{otherwise}
\end{cases} \tag{21}
\]

Before we move on to the derivation of equilibrium in a cooperative organization it is useful to quickly look at some properties of the equilibrium effort levels and profits. First of all, equilibrium effort levels \(\hat{e}_1(v^*, k^*), \hat{e}_2(v^*, k^*)\) as given in (16) and (17) are independent of the profitability of the outside opportunity \(r\). The reason for this is that in both cases, the optimal spread between first and second prize \((2k^* - 1)v^*\) is independent of any characteristic of the outside opportunity. This implies that even in presence of a binding IR-constraint there exists some effort level and subsequently a constant revenue level which the league wishes to attain. For \(\beta > \frac{1}{2}\), the spread is higher whenever investments in the outside opportunity occur.

On the contrary, both clubs’ expected equilibrium profits are non-decreasing in the productivity of outside-opportunity investments. This result is driven by the fact that the league is forced to make club 2 indifferent between joining the league and choosing its outside opportunity. Increased productivity of outside opportunity investments c.p. increases club 2’s profits when going into the outside opportunity. And even though all involved parties know that club 2 will never choose its outside opportunity in equilibrium, the league still has to adjust to the increase in outside-option profitability by increasing contest-prize money as well as the share of prize money awarded to the loser. The league does so while keeping the spread constant in order to achieve its desired revenue level.

Next, consider how the degree of asymmetry, as represented by the parameter \(\beta\), affects equilibrium profit levels. Differentiating the respective expressions yields \(\frac{\partial}{\partial \beta} E(\pi_1) < 0, \frac{\partial}{\partial \beta} E(\pi_2) > 0\). A higher degree of asymmetry increases club 1’s profits. This is on the one hand due to the fact that, as has been seen

\[8\text{Recall that } \beta \in (\frac{1}{2}, 1) \text{ and that the degree of heterogeneity is decreasing with } \beta.\]
above, a higher degree of asymmetry increases club 1’s equilibrium effort thus yielding an increased probability of winning the championship and subsequently increasing its expected profits. On the other hand, in case club 2’s IR-constraint is binding, club 2 must be compensated by the league for the lowered probability of success resulting from the increased degree of asymmetry. The league does so by reducing the spread.\(^9\) In this case, the combined effect of lowered costs and higher probability of success outweighs the dampening effect of the reduced spread on to the expected profits of club 1.

Regarding the profits of club 2, it is straightforward that $\beta$ does not have any effect on profits of club 2 in case club 2 invests into its outside opportunity. The less efficient club will always be left with $\pi_O = \frac{r_2}{r}$, the profit it can guarantee itself via outside opportunity. In the case in which club 2 does not invest into the outside opportunity, an increase of the degree of asymmetry lowers its expected profits because it lowers its equilibrium probability of success.

An additional point worth noting is that $\frac{\partial}{\partial e} E(W_M) < 0$, that is, expected aggregate welfare is increasing with an increasing degree of asymmetry. In case of no investments into the outside opportunity on behalf of club 2, the reason is straightforward. A higher degree of asymmetry is equivalent to lower effort costs of club 1 and subsequently, as discussed above, higher profit levels both for club 1 as well as the league. These increases are higher than the resulting negative difference in profit of club 2. In case of outside opportunity investments on behalf of club 2, the higher degree of asymmetry reduces equilibrium efforts because club 2 anticipates that the league will appropriate a major part of the increased revenue. However, expected profits of club 1 increase by a large amount via reduced effort costs, such that the reduced profits both of club 2 as well as the league are compensated for.

3.2 Cooperative Organization

Let us now suppose that the league is constituted as a cooperative of the two clubs. This is a situation similar to the U.S. Major Leagues most of which are organized in a manner resembling cooperatives. Every club-owner accounts for one vote and all major issues are decided by majority if not unanimous agreement. Due to the loss of ‘market interaction’ and the subsequent absence of an independent profit-maximizer, this organization may be superior in terms of welfare to the setting described above. Most importantly, the fact that every participant can cast a vote and thus affect league matters renders the outside opportunity irrelevant. Investing into the outside opportunity in order to extract rents from the league will hurt the league and therefore the clubs themselves which - in this setting - constitute the league. Thus, outside opportunity investments amount to taking money out of the own pocket and will therefore not be considered as an alternative.

We will subsequently assume that all of the league revenue $R(e_1, e_2)$ is dis-

\(^9\)It can be shown that a higher degree of asymmetry reduces the equilibrium spread in case of investments into the outside opportunity.
tributed among the two clubs, i.e. administrative costs on behalf of the league amount to zero. For the moment we will assume that the fraction $\kappa \in \left[ \frac{1}{2}, 1 \right]$ of total revenue is awarded to the champion. Then, the clubs expected profits are given by

$$E(\pi_1) = p_1 \kappa R(e_1, e_2) + p_2 (1 - \kappa) R(e_1, e_2) - \beta c_1 e_1$$
$$E(\pi_2) = p_2 \kappa R(e_1, e_2) + p_1 (1 - \kappa) R(e_1, e_2) - c_1 e_2$$

where the probabilities of success $p_i$ remain unchanged from above, i.e. $p_i = \frac{e_i}{e_1 + e_2}$, $i = 1, 2$. The Nash Equilibrium of this contest is determined by the following FOC

$$\frac{\partial E(\pi_1)}{\partial e_1} = 0$$
$$\frac{\partial E(\pi_2)}{\partial e_2} = 0$$

yielding the following equilibrium effort levels

$$(\hat{e}_1(\kappa), \hat{e}_2(\kappa)) = \left( \frac{(4\kappa - 1)^2 (\kappa(3 - \beta) - 1)}{4c^2(2k - 1)(1 + \beta)^3}, \frac{(4\kappa - 1)^2 (\kappa(3\beta - 1) - \beta)}{4c^2(2k - 1)(1 + \beta)^3} \right)$$

Note that $\hat{e}_2(\kappa) \geq 0$ for $\kappa \geq \frac{\beta}{2\beta - 1}$.

Once the two clubs are organized in a cooperative manner, the problem of allocating the decision rights arises. Since the focus of this paper does not lie on decision processes in cooperatives but on the allocative superiority of one organizational form versus another we will not enter this discussion and suppose that the sharing parameter $\kappa \in \left[ \frac{1}{2}, 1 \right]$ is chosen by some independent commissioner such that total revenue $R(c_1, e_2)$ is maximized. The commissioner then solves

$$\max_{\kappa} \left\{ (\hat{e}_1(\kappa) + \hat{e}_2(\kappa))^{\frac{1}{2}} \right\} \quad \text{s.t. } \kappa \in \left[ \frac{1}{2}, 1 \right]$$

The solution to the above problem is given by $\kappa^* = 1$.\textsuperscript{10} Thus, as in the market-interaction setting in the case in which club 2’s IR-constraint is not binding, in order to maximize revenue it is optimal to stage a ‘winner-takes-all’ contest. Even though clubs are organized as a cooperative, it is desirable from the viewpoint of a revenue-maximizer to award total revenue to the winner. This result however is sensitive to the specific nature of the revenue function on the one hand and the fact that revenue rather than joint profits are maximized. The fact of the matter though is that all revenue is passed back to the teams. The clubs’ equilibrium effort levels in the presence of $\kappa = \kappa^* = 1$ are then given by

$$(\hat{e}_1(\kappa^*), \hat{e}_2(\kappa^*)) = \left( \frac{9(2 - \beta)}{4c^2(1 + \beta)^3}, \frac{9(2\beta - 1)}{4c^2(1 + \beta)^3} \right)$$

\textsuperscript{10}See appendix A.3 for a proof.
Plugging these effort levels into (22) and (23) and rearranging terms yields expected profits in equilibrium conditional on $\kappa = \kappa^* = 1$:

$$E(\pi_1^C) = \frac{3(\beta - 2)^2}{4c_1(1 + \beta)^3}$$

(29)

$$E(\pi_2^C) = \frac{3(2\beta - 1)^2}{4c_1(1 + \beta)^3}$$

(30)

Expected welfare as measured by aggregate profits is then given by

$$E(W_C) = E(\pi_1^C) + E(\pi_2^C) = \frac{3[(\beta - 2)^2 + (2\beta - 1)^2]}{4c_1(1 + \beta)^3}$$

(31)

As in the market-interaction setting, it is useful to quickly look at the impact of team-asymmetry onto equilibrium effort and profit levels. It can be shown that $\frac{\partial \pi_1(\kappa^*)}{\partial \beta} < 0$ and $\frac{\partial \pi_2(\kappa^*)}{\partial \beta} > 0$. Here, the same mechanisms as in the market-interaction case are at work: A lower $\beta$ is equivalent to reduced marginal costs of club 1 leading to increased effort. As a reaction to this increased effort level of club 1, club 2 lowers its own effort to again equalize marginal revenue and marginal costs. More interestingly, it is the case that $\frac{\partial}{\partial \beta} E(\pi_1^C) < 0$ while for $\beta \in \left(\frac{1}{2}, 1\right)$ it is the case that $\frac{\partial}{\partial \beta} E(\pi_2^C) > 0$, i.e. a higher degree of asymmetry decreases club 2’s profits. Analogously, the same argumentation as in the previous section and the case in which club 2 does not invest into the outside opportunity applies.

### 3.3 Comparison

Having derived the equilibrium profits in both institutional settings, the natural question that arises is, which of the settings is better. We will do so by comparing expected welfare across the two settings. It can be shown that $E(W_C) \geq E(W_M)$ rendering the cooperative organization desirable from a social point of view.\(^\text{11}\) The reasons lie first and foremost in the organizational differences between the two settings. If relations between clubs and the league are governed by the market, the profit-maximizing league passes some share $v < R(e_1, e_2)$ to the clubs. Because the prize money and subsequently the spread between first and second prize is lower than in the cooperative setting, effort levels will be lower resulting in a suboptimal small total surplus. Additionally, whenever the outside opportunity is sufficiently profitable, club 2 will exert his bargaining power in order to appropriate a larger share of the pie. These rent-appropriation measures occur in form of investments into the outside opportunity which do not come for free. From an allocative point of view, this distributional fight is inefficient since it does only alter the distribution but

\(^{11}\)A technical assumption needed to ensure that indeed the cooperative regime is desirable for all $\beta$ is that $r^2c_1 \geq \frac{1}{2}$. However, it can be shown that if the degree of asymmetry is sufficiently high (i.e. $\beta < 0.9$) then aggregate profits are always higher in the cooperative setting.
is totally unproductive. But not only are these rent-appropriation measures inefficient in the sense that they do not increase total surplus. Furthermore, they induce a suboptimal high spread between first and second prize on behalf of the league.\textsuperscript{12} This again distorts the incentives of both of the clubs and causes surplus to decline even further.

Summarizing the results derived in the preceding subsections, we can state that a cooperative organization possesses two major advantages over a regime governed by the marketplace, one of which also holds in cases in which the clubs’ outside opportunities are a priori unattractive and the threat of league exit does not exist. Firstly, the fact that the league acts as an independent agent maximizing the residual, distorts incentives for the clubs interacting in the league. Secondly, whenever the outside opportunity is sufficiently profitable, the less profitable club will exert its bargaining power in order to appropriate a larger share of the pie. Thus, from a welfare point of view, a cooperative organization of professional sports leagues is desirable.

4 Discussion

As has been stated in the introduction, considerable differences exist between European and North American professional sports leagues both in terms of profitability and in terms of the preferred organizational regime for sports leagues. Another stylized fact is that in several European leagues - most notably Formula One motor racing - there exist endeavours on behalf of clubs to increase pressure on the league body in order to accumulate a larger share of total surplus. We have provided a game-theoretic model showing that when a profit-maximizing league body and asymmetric clubs coordinate their activities - i.e. the staging of the championship - via the market, two mechanisms exist that lead to a lower level of aggregate profits. Firstly, the existence of a profit-maximizing intermediary provides the clubs with inefficient incentives when engaging in the league. Secondly, the absence of alternative sources of income for the league body endows some subset of clubs with considerable bargaining power which will be exerted if the outside option is sufficiently profitable. A remedy for these problems is to unite all agents under one legal entity. We believe that the cooperative is the most favorable form for this entity. This is due to the fact that the clubs remain independent but are still able to exert influence over matters that affect the league as a whole. Basically, a merger of all clubs into one ‘league-corporation’ is possible, too. We believe however, that this would raise other unfavorable issues. First of all, if clubs are not independent but are united under one corporate ‘roof’, one would have a hard time convincing consumers of the integrity of the championship race, since the league owner possessed strong incentives to distort if not the championship as a whole then single games, into his favor. But - as recent events in Germany have shown - integrity of the championship race is one of the main pillars of consumer satisfaction with the

\textsuperscript{12}It can be shown that the spread is higher in the equilibrium in which there are investments into the outside opportunity than in the equilibrium without these investments.
product 'professional sports championship'. However, there exist leagues under single ownership such as World Wrestling Entertainment (WWE) in which consumers seem to accept the fact that championships are 'fixed'. It is ambiguous though, whether this can still be regarded as a sport or - what seems to be more appropriate - a generic form of entertainment. A second problem which arises in a 'corporate league' is incentivizing local team-managers. If investments are specific by nature, then setting correct incentives for local team managers might be a difficult task. A cooperative organization however circumvents these issues by leaving the clubs independent.

Stylized facts support the results of our model, as the U.S. Major Leagues seem to operate much more profitably than their European counterparts. Apart of the organizational issues raised in this paper, mechanisms lowering expenses such as salary caps might contribute to an increased profitability. A sound empirical analysis of the determinants of financial success in professional sports leagues is subject to future research. Nonetheless, as experiences in the European top soccer leagues show, the implementation of mechanisms such as salary caps poses much lesser problems in a league organized as a cooperative. Thus, even if it were the case that a substantial part of the financial performance of U.S. Major Leagues stemmed from salary caps or luxury taxes it would be a good measure to consider a cooperative organization of the league since this would facilitate the introduction of such measures.

References


A Appendix

A.1 Proof of Proposition 1

The Lagrange function of problem (10) is given by

\[ L = R - v + \lambda \left( \frac{\beta}{c_1(1 + \beta)} \right)^{-\frac{1}{2}} \left( \frac{2k - 1}{c_1(1 + \beta)} \right) - 1 \]

where \( \lambda \) and \( \omega \) are the multipliers on the IR-constraint and the constraint concerning \( k \).

For notational simplicity let \( r_{c_2}^{0.5} = a \). Plugging the respective expressions into (32) and rearranging terms yields

\[ L = R - v + \lambda \left[ \frac{v [\beta - 1] k + 1}{1 + \beta} \right] - \frac{\beta v (2k - 1)}{(1 + \beta)^2} - a \] + \( \omega (1 - k) \) \hspace{1cm} (33)

Then, the Kuhn-Tucker conditions are given by

\[ \frac{\partial L}{\partial v} = 1 \left( \frac{(2k - 1)v}{c_1(1 + \beta)} \right)^{-\frac{1}{2}} \left( \frac{2k - 1}{c_1(1 + \beta)} \right) - 1 \]

\[ + \lambda \left[ \frac{\beta k + 1 - k}{1 + \beta} - \frac{\beta (2k - 1)}{(1 + \beta)^2} \right] = 0 \] \hspace{1cm} (34)

\[ \frac{\partial L}{\partial k} = 1 \left( \frac{(2k - 1)v}{c_1(1 + \beta)} \right)^{-\frac{1}{2}} \left( \frac{2v}{c_1(1 + \beta)} \right) - \omega + \lambda \left[ \frac{\beta v - v}{1 + \beta} - \frac{2\beta v}{(1 + \beta)^2} \right] \]

\[ = 0 \]

\[ \lambda \left[ \frac{v [(\beta - 1) k + 1]}{1 + \beta} - \frac{\beta v (2k - 1)}{(1 + \beta)^2} - a \right] = 0 \] \hspace{1cm} (36)

\[ \omega (1 - k) = 0 \] \hspace{1cm} (37)

The solution will be derived by breaking the above system into several subcases.

- \( E(\pi_2) > a - z_2 \Rightarrow \lambda_1 = 0 \)
  - \( \omega_1 > 0 \Rightarrow k_1 = 1 \)

Plugging the respective values into (34) and (35) and solving this reduced system for \( v \) and \( \omega \) yields

\[ (v_1, k_1, \lambda_1, \omega_1) = \left( \frac{1}{4c_1(1 + \beta)}, \frac{1}{2c_1(1 + \beta)} \right) \] \hspace{1cm} (38)

\[ ^{13} \text{Note that there is no nonnegativity constraint concerning } k \text{. For reasons of simplicity this constraint has not been added. However, we will check whether the constraint is satisfied.} \]
Note however, that this solution holds only conditional on $E(\pi_2) > a - z_2$, that is, it only constitutes a solution if
\[
\frac{v_1 (1 + \beta) k_1 - 1}{1 + \beta} - \frac{\beta v_1 (2k_1 - 1)}{(1 + \beta)^2} - a \frac{[(1 + \beta) - 1]}{4c_1 (1 + \beta)^2} - \frac{\beta v_1 (2k_1 - 1)}{(1 + \beta)^2} > a
\]
\[
\Leftrightarrow \frac{[(1 + \beta) - 1]}{4c_1 (1 + \beta)^2} - \frac{\beta}{4c_1 (1 + \beta)^3} > a
\]
\[
\Leftrightarrow a_0 = \frac{\beta^2}{4c_1 (1 + \beta)^2} > a
\]

\(- k < 1 \Rightarrow \omega = 0 \)

Substituting the respective values into (34) and (35) and solving this reduced system for $v$ and $k$ yields $(v, k) = (0, \frac{1}{2})$. However, this cannot be a solution because it must be the case that $E(\pi_2) > a - z_2$, which is violated.

\(- k = 1, \omega = 0 \)

Plugging these respective values into (34) and (35) reveals that for (34) to hold it must be the case that $v > 0$ while for (35) to be satisfied we must have $v = 0$. Hence, there is no solution in this case.

- $E(\pi_2) = a - z_2 \Rightarrow \lambda \geq 0$

- $\omega_2 \geq 0 \Rightarrow k_2 = 1$

Under these assumptions, the above system (34) - (36) reduces to
\[
\frac{1}{2} \left( \frac{v_2}{c_1 (1 + \beta)} \right)^{-\frac{1}{2}} \frac{1}{c_1 (1 + \beta)} - 1 + \lambda_2 \left( \frac{\beta}{1 + \beta} - \frac{\beta}{(1 + \beta)^2} \right)
= 0
\]
\[
\frac{1}{2} \left( \frac{v_2}{c_1 (1 + \beta)} \right)^{-\frac{1}{2}} \frac{2v_2}{c_1 (1 + \beta)} - \omega_2
\]
\[
+ \lambda_2 \left( \frac{\beta v_2 - v_2}{1 + \beta} - \frac{2\beta v_2}{(1 + \beta)^2} \right) = 0
\]
\[
\frac{\beta v_2}{1 + \beta} - \frac{\beta v_2}{(1 + \beta)^2} - a = 0
\]
Solving this system for \((v_2, \lambda_2, \omega_2)\) yields

\[
\begin{align*}
v_2 &= \frac{a(1 + \beta)^2}{\beta^2} \\
k_2 &= 1 \\
\lambda_2 &= 1 + \frac{1}{\beta^2} + \frac{2}{\beta} - \left(\frac{a(1 + \beta)}{c_1\beta^2}\right)^\frac{1}{2} \\
\omega_2 &= \frac{\beta^2(1 + \beta)^2}{2\beta^4} \left[ \beta^2 \left(\frac{a(1 + \beta)}{c_1\beta^2}\right) + 2a (\beta^2 - 2\beta - 1) \right]
\end{align*}
\]

However, this solution has been derived under the assumption that \(\lambda_2 \geq 0\) and \(\omega_2 \geq 0\). Rearranging (44) and (45) yields

\[
\begin{align*}
\lambda_2 &\geq 0 \Leftrightarrow a \geq \frac{\beta^2}{4c_1(1 + \beta)^3} = a_0 \\
\omega_2 &\geq 0 \Leftrightarrow a \leq \frac{\beta^2(1 + \beta)}{4c_1(\beta^2 - 2\beta - 1)^2} = a_1
\end{align*}
\]

\(- k_3 < 1 \Rightarrow \omega_3 = 0\)

Plugging these values into (34) - (36) and solving the resulting sub-system for \((v_3, k_3, \lambda_3)\) yields

\[
\begin{align*}
v_3 &= 2rz_2^\frac{1}{2} + \frac{1 + \beta}{4c_1 \left(1 + 2\beta - \beta^2\right)} \\
k_3 &= \frac{1 + 3\beta + 2\beta^2 + 4c_1 a (\beta^2 - 2\beta - 1)^2}{\left(1 + 2\beta - \beta^2\right) \left[1 + \beta - 8c_1(\beta^2 - 2\beta - 1)\right]} \\
\lambda_3 &= 2
\end{align*}
\]

However it must be the case that \(k_3 < 1\) which is equivalent to

\[
a > \frac{\beta^2(1 + \beta)}{4c_1(\beta^2 - 2\beta - 1)^2} = a_1
\]

\(\blacksquare\)

### A.2 Proof of Proposition 2

As has been shown above in the proof of proposition 1, for \(a(z_2) \geq a_0\) club 2’s IR-constraint will always be binding, i.e. when joining the league, club 2 will always be as well off as in his outside opportunity. Therefore, when deciding whether to invest in the outside opportunity, club 2 will compare profits out of doing so to profits in case of \(z_2 = 0\). When maximizing outside opportunity profits, club 2 solves

\[
\max_{z_2} \left\{ rz_2^{0.5} - z_2 \right\}
\]

22
yielding $z_2^* = \mathbb{E}z_2 = r(z_2^*)^{0.5} - z_2^* \equiv \pi_O$. Note that this is also exactly what club 2 will receive when participating in league play and $a(z_2^*) = a\left(\frac{r^2}{T}\right) = \frac{r^2}{T} \geq a_0 = \frac{\beta^2}{4c_1(1+\beta)^3}$. The alternative for club 2 is not to invest in the outside opportunity, i.e. $z_2 = 0$, leading to a prize structure of $(\hat{k}, \hat{\nu}) = \left(1, \frac{1}{4c_1(1+\beta)}\right)$ and following expected profits:

$$E(\pi | z_2 = 0) = \hat{\nu}_2 \hat{k} \hat{\nu} - c_1 \hat{e} \left(\hat{\nu}, \hat{k}\right) = \frac{\beta}{1 + \beta} \frac{1}{4c_1(1 + \beta)} - \frac{\beta}{4c_1(1 + \beta)^3} = \frac{\beta^2}{4c_1(1 + \beta)^3}$$

Therefore, club 2 will always invest into the outside opportunity if

$$\pi_O \geq E(\pi | z_2 = 0) \iff \frac{r^2}{4} \geq \frac{\beta^2}{4c_1(1 + \beta)^3} \iff r^2 \geq \frac{\beta^2}{c_1(1 + \beta)^3}$$

Note that if condition (55) is violated, club 2 will always choose $z_2 = 0$ since $\frac{r^2}{T}$ is the maximum profit club 2 can achieve using his outside opportunity. Note also that club 2 will always join the league since it is assumed that in the case of indifference on behalf of club 2, club 2 will join the league.

### A.3 Solution of Problem (27)

First of all, note that the solution of problem (27) is will also be the solution when maximizing $\hat{e}_1(\kappa) + \hat{e}_2(\kappa)$. Then, the Lagrangian is given by

$$\mathcal{L} = \hat{e}_1(\kappa) + \hat{e}_2(\kappa) + \lambda \left(\kappa - \frac{1}{2}\right) + \gamma(1 - \kappa)$$

where

$$\hat{e}_1(\kappa) + \hat{e}_2(\kappa) = \frac{(4\kappa - 1)^2 [\kappa(2\beta - 2) - (1 + \beta)]}{4c_1^2(2\kappa - 1)(1 + \beta)^3}$$

The Kuhn-Tucker constraints are given by

$$\frac{\partial \mathcal{L}}{\partial \kappa} = \frac{8\kappa - 2}{c_1^2(1 + \beta)^2} + \lambda - \gamma = 0$$

$$\lambda \left(\kappa - \frac{1}{2}\right) = 0$$

$$\gamma(1 - \kappa) = 0$$

Once again, we will analyze the problem by looking at several subcases.

- $\gamma > 0 \implies \kappa = 1 \implies \lambda = 0$
Then, condition (59) reduces to
\[
\frac{6}{c_1^2(1 + \beta)^2} = \gamma
\] (62)
which is a solution.

- $\kappa = \frac{1}{2} \implies \lambda \geq 0, \lambda = 0$
  Then, condition (59) is given by
  \[
  \frac{2}{c_1^2(1 + \beta)^2} = -\lambda
  \] (63)
which yields a contradiction since the left hand side is positive.

- $\kappa \in \left(\frac{1}{2}, 1\right) \implies \lambda = \gamma = 0$
  In this case, condition (59) is given by
  \[
  \frac{8\kappa - 2}{c_1^2(1 + \beta)^2} = 0
  \] (64)
yielding a contradiction since the left-hand side is positive for $\kappa \in \left(\frac{1}{2}, 1\right)$.

Therefore, the unique solution of problem (27) is given by $\kappa = 1$. 

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